DISSERTATION

ATMOSPHERIC WATER BALANCE OF THE UPPER COLORADO RIVER BASIN

Sulomitted by

James L. Rasmussen

In partial fulfillment of the requirements for the Degree of Doctor of Philosophy Colorado State University Fort Collins, Colorado

January, 1968

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COLORADO STATE UNIVERSITY

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ABSTRACT OF DISSERTATION

ATMOSPHERIC WATER BALANCE OF THE UPPER COLORADO RIVER BASIN

The atmospheric branch of the hydrologic cycle is investigated to determine the wintertime accumulation of water over the Upper Colorado River Basin. The parameter precipitation minus evaporation is computed as a residual from the atmospheric water balance equation. The study covers the seven winter seasons 1957 through 1963.

The results show that the periods of evaporation as well as the periods of heavy precipitation determine the seasonal water balance of the basin. The seasonal course of daily evaporation rate is determined. The evapcration rate varies by a factor of two over the winter season. Further, a strong decay with time of evaporation rate is observed during the early and mid-winter months. A less pronounced decay is obtained during March and April.

The basin precipitation data obtained from the atmospheric water balance computation are compared to a basin precipitation estimate independently obtained using data from fourteen rain gauges. The conclusion is reached that the gauge data underestimate the basin precipitation by about fifty per cent. Much of this bias is shown to be due to the lack of sampling over the high elevation regions where the precipitation is greatest.

The wintertime accumulation of water over the basin is shown to be highly related to the April through March runoff from the basin. The relationship shows that the accumulated water is apportioned by a ratio of one to four between runoff and evaporation respectively. Finally the application of the atmospheric water balance computation to the problem of runoff forecasting is discussed.

> James Laurence Rasmussen Department of Atmospheric Science Colorado State University January, 1968

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CHAPTER I INTRODUCTION

Purpose

The annual runoff from the Colorado River Basin varied by more than a factor of five over the seven water-years 1957^1 through 1963. This extreme var ability causes serious difficulty for the arid southwest United States, a large portion for which the Colorado River is the major source of water supply. It is of interest, therefore, to understand the factors causing this variability of the water yield. These factors are precipitation and evaporation. The annual flow of the Colorado River is largely derived from the melt of snow accumulated during the winter season over the high elevation regions of the headwaters of the Colorado River and its tributaries the Green and San Juan Rivers. Studies by Marlatt and Riehl (1963) and Riehl and Elsberry (1964) describe the winter and annual precipitation regime of the Colorado Bas_n as being dominated by the occurrence of large precipitation episodes separated by periods of little or no precipitation and undoubtedly significant evaporation, even in winter. In this paper the nature of, and roles played by, the evaporation periods as well as the storm periods in the water budget of the Colorado River are studied for the seven winters 1957 through 1963. The purpose of this study is to answer the questions:

A water-year is defined as beginning on 1 October of the year before record and ending on 30 September of the year of record. The winter season is defined as the period October through April and the summer season as May through September.

- What is the amount of water accumulated over the Colorado River watershed during the winter season and what is the relationship of this accumulation to the annual discharge from the basin?
- 2) What are the roles played by the precipitation and evaporation periods in this accumulation?
- 3) What are the synoptic-scale meteorological conditions associated with both the evaporation and precipitation periods?

Background

Traditionally, studies of the hydrologic balance of river basins have been apprcached from the point of view of the terrestrial part of the hydrologic cycle. The factors determining the runoff from an area are precipitation, evaporation, change in water storage and underground seepage from the basin. Such an approach to the study of hydrologic problems is often plagued by measurement deficiencies. Runoff is measured the most satisfactorily of all the variables; however, the runoff from large mountainous regions integrates the water accumulated over both space and time so that the effect on the runoff from a shorter period within the integrated period cannot be ascertained. Meaningful evaporation measurements are most difficult to make and direct measurement methods require a sophisticated laboratory. Sellers (1965) gives a good review of the various techniques available for direct measurements of evaporation as well as indirect methods relying on climatological data and semiempirical formulation. Precipitation gauge measurements are wellknown to be biased toward the low side (Weiss and Wilson, 1957) and this bias becomes extreme in the measurement of snow. As the size of the area for which one seeks data representation increases, the measurement problem increases. If one deals with a large

mountainous region, the measurement problem is maximized because for such regions not only is the density of observations small but they are typically biased toward the lower elevations. The net result of these problems has been slow progress in understanding the hydrology of large mountainous regions.

Alternately, the atmospheric part of the hydrologic cycle may be studied to evaluate the net deposition of water over an area. A budget parallel to that of the terrestrial part of the hydrologic cycle must be observed. The atmospheric water balance may be expressed as the evaporation minus precipitation occurring over an area balanced by the net transfer of water mass through the atmospheric volume over the area and the change in storage of water mass within the atmospheric volume. In theory then, given a continuous distribution in time and space of the atmospheric water mass, an accounting can be done to determine, as a residual, the quantity evaporation minus precipitation. In practice, however, the distribution of water in the atmosphere is not continuously known but rather only the water in the vapor state is sampled and at time intervals of twelve hours and over distances of hundreds of kilometers. The problem then is to approximate the water balance from this imperfect sampling procedure, realizing that the computation is only meaningful over sufficiently large areas and for sufficiently large weather systems.

This paper summarizes the methodology and results of research applying the atmospheric water balance approach to study some of the hydrologic features of the Colorado River Basin in an effort to answer the questions posed in the preceding section.

Review of Atmospheric Water Balance Investigations

The role of the atmosphere in the hydrologic cycle has been studied primarily on the scale of the general circulation. Starr

and White (1955), Starr, Peixoto and Livados (1958) and Starr and Peixoto (1957) have computed the meridional and zonal fluxes and the flux divergence of water vapor on a global scale for the calendar year 1950. Studies on this scale are particularly applicable to the evaluation of the contribution to the atmospheric heat balance by the transport and release of latent heat and its relationship to the general circulation of the atmosphere. The above studies followed an initial work by Benton and Estoque (1954) in which the atmospheric water balance for the North American Continent during the calendar year 1949 was evaluated. This study yielded monthly and annual values of evaporation minus precipitation for the entire continent and were found to be in general agreement with hydrologic measurements. The above studies were gross in their horizontal and vertical resolution and were not intended to be applied to areas of the scale of an individual watershed. Hutchings (1961) estimated evaporation minus precipitation for Australia during the year 1956 using the atmospheric water balance technique. His annual result was also in agreement with independently obtained estimates.

Recently Rasmusson (1966) computed the atmospheric water balance for the North American Continent and for regions within the continent. Eis study covered a two-year period, May, 1961, through April, 1963. He used the evaporation minus precipitation obtained from the atmospheric water balance computations and the observed runoff from various regions to determine the annual change in storage of ground water over the regions. He further investigated possible sources of error in the computation and concluded that a major source of error is due to the diurnal variation in the wind field. This error arises from the fact that sampling the atmosphere twice daily does not sufficiently define this diurnal variation and thus, a systematic error may contaminate the computation. Based on this error analysis, Rasmusson defines a lower limit to the area

over which reliable results on a monthly to annual basis can be obtained. The limiting size of the area according to this analysis is 10^6 km^2 . On the other hand, Hutchings (1957), Väisänen (1962), Palmén and Söderman (1966), and Bradbury (1957), among others, have obtained quite reasonable and independently confirmed results for much smaller areas and/or for much shorter periods of time. These studies have been aimed at quite different problems; from the measurement of evaporation and evapotranspiration in the cases of Palmén and Söderman (1966) and Väisänen (1962) to the water budget of individual storm systems in the case of Bradbury (1957). These studies show that a careful atmospheric water balance computation can be done for areas of size $3 \times 10^5 \text{ km}^2$ and over periods of less than one month.

A comprehensive review of the methodology and problems one faces in the computation of the atmospheric water balance is given by Palmén (1967). In addition, this monograph outlines the progress made over the last twenty years in the study of the water balance of the atmosphere and also outlines proposals for further action.

No single study mentioned above covered a period of more than two consecutive years and nothing has been done solely for an area comprised of one hydrologically well-documented watershed. It is hoped that the study reported herein will help to fill this void.

The Colorado River Basin

The Colorado River Basin (Figure 1) drains an area of approximately 6.3x10⁵ km² of seven states. The important runoff comes from the melt of snow in the high elevations of the headwaters of the Colorado River and its tributaries, the Green and San Juan Rivers. The drainage area of these rivers has been historically referred to as the Upper Colorado River Basin. For the purposes of this report, the Upper Basin is reckoned from the river gauging



Figure 1. The Colorado River Basin.

station at Lee's Ferry, Arizona, (Figure 2) and covers an area of $2.6 \times 10^5 \text{ km}^2$.

The topography of the Upper Colorado Basin is dominated by high mountain ranges on most of its periphery except along the southern border and a relatively low saddle on the northeast border. A highly smoothed topography is shown in Figure 2. Table 1 lists the percent distribution of surface area of the basin in various elevation classes. A relatively small percentage of the total area is, however, the source region of the major portion of the annual river flow at Lee's Ferry.

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Percent of the Area of the Upper Colorado River Basin Classed According to Elevation Above Sea Level:

	> 11,000	8,000-	5,000-	
Elevation range (ft)		11,000	8,000	< 5,000
Percent area	3	24	63	10

A major climatological feature of the Upper Colorado River Basin is the large variability of precipitation. Marlatt and Riehl (1963) have shown that the annual precipitation over the Upper Colorado River Basin varied by a factor of 2 over the period 1930 to 1960. The runoff at Lee's Ferry showed even greater variability, a factor of 5 over the same period (Yevdjevich, 1961). This amplification of the variability from precipitation to runoff underscores the arid nature of the region. Indeed, over most of the region the potential evaporation greatly exceeds the precipitation and the resulting stream flow from small local watersheds is ephemeral in nature, lasting only a short time after a precipitation occurrence. Only in the high elevation is the precipitation great enough and the potential evaporation low enough to sustain streamflow continuously



Figure 2. The upper Colorado River Basin above Lee's Ferry, Arizona. The highly smoothed topography in units of 1000's of feet msl. The course of the Colorado (center), Green (left), and San Juan (right) rivers are shown. (McDonald, 1960). The large fluctuations in the annual riverflow of the Colorado River have given rise to the planning and the construction of large water storage facilities so that the fluctuations in the riverflow can be artificially controlled and hence more useful for agricultural, industrial, and domestic purposes. The limit of such construction is dictated by the amount of water available and its variation over long time periods.

Over a long period of time in arid regions, the evaporation from a water surface is greater than from a soil surface (Sellers, 1965). The soil surface dries with time, thus inhibiting evaporation. The continuing construction of surface storage facilities, therefore, can be detrimental to some degree to the water balance of the basin. The increase of surface area of reservoir water allows for an increase in evaporation with no corresponding increase in precipitation. Care must be taken so that the optimum use of the stored water is made and that the evaporation from the reservoirs is held at a level that is rot detrimental to the water balance.

The use of the Colorado River waters is regulated by several documents of which the most important is the Colorado River Compact of 1922. This document requires the Upper Basin to proivde an average discharge² of 3.6 cm to the area below Lee's Ferry. This required discharge is over half the average annual discharge, 6.4 cm per year. Complicating this picture are the continued depletions for municipal and irrigation uses within the Upper Colorado River Basin and also trans-mountain diversions from the basin. Yevdjevich (1961) shows that the current annual depletions are about

^{2.} The term discharge as used here is the annual rate of flow of the river. The measure of discharge employed in this paper is commonly called "unit yield" and represents the depth the water would stand if all the runoff were spread uniformly over the whole watershed. For the Upper Colorado River Basin, a unit yield of 1 cm corresponds to almost 2 million acre-feet of water.

1.0 cm per year, and Riter (1956) estimates that an additional 1.2 cm per year will be depleted by existing and authorized projects in the future. These current and anticipated demands (2.2 cm per year) along with the required delivery at Lee's Ferry (3.6 cm per year) amount to 90 percent of the average annual discharge. An extended period of drought could have disastrous consequences for a river basin under such a delicate balance between supply and demand. Massive industrial developments (e.g., oil shale development) could invoke demands for water which also would upset the balance.

It is imperative, therefore, that the hydrology of the Colorado River Basin be understood in detail so that these problems are faced from the vantage point of firm scientific knowledge. It is hoped that this paper will provide some of the background necessary for future planning.

CHAPTER II

METHOD

The objectives of this work may be attained by the determination of the exchange of water and water vapor at the earth-atmosphere interface of the Upper Colorado River Basin through the observation of the spacial and time distributions and changes of water and water vapor in the atmosphere over the basin. The exchange at the earth's surface must be the evaporation minus the precipitation. The evaporation alone may then be obtained providing the precipitation is known.

As in most meteorological investigations, the observational material is not complete. The findings to be presented herein are to a large part based on residuals of computations and, therefore, subject to error. This problem is minimized, however, due to the availability of independent measurements of some of the calculated quantities, and these checks were employed wherever possible.

The Atmospheric Water Balance

Let us consider a parcel of air having a specific humidity, q, and a ratio of mass of water (liquid or ice) to mass of moist air r. In a coordinate system with pressure, p, as the vertical coordinate, x as distance eastward, y as distance northward, the time rate of change of water and water vapor written in terms of local derivatives is:

 $\frac{d}{dt} (q + r) = \frac{\partial(q)}{\partial t} + \frac{\partial(r)}{\partial t} + IV_2 \cdot \nabla_2 q + IV_2 \cdot \nabla_2 r + \omega \frac{\partial q}{\partial p} + \omega \frac{\partial r}{\partial p}$ (1) where t is time, IV_2 and ∇_2 are the velocity vector and gradient operator on a pressure surface respectively, and ω is $\frac{dp}{dt}$. Let us further assume that there is no water in any phase being created or destroyed through chemical processes within the parcel. Substituting the equation of mass continuity

$$\nabla_2 \cdot IV_2 = -\frac{\partial \omega}{\partial p}$$
(2)

one obtains:

 $\frac{d(q+r)}{dt} = \frac{\partial q}{\partial t} + \frac{\partial r}{\partial t} + \nabla_2 \cdot IV_2q + \nabla_2 \cdot IV_2r + \frac{\partial(\omega q)}{\partial p} + \frac{\partial(\omega r)}{\partial p}$ (3) Let us define an increment of mass as $\delta m = \delta \times \delta y \frac{\delta p}{g}$ where g is the acceleration of gravity. Integrating (3) over the mass of an atmospheric column extending from the earth's surface to some level in the free atmosphere one obtains:

$$0 = \int_{\delta m} \frac{\partial}{\partial t} (q) \, \delta m + \int_{\delta m} \frac{\partial (r) \delta m}{\partial t} + \int_{\delta m} \nabla_2 \cdot IV_2 q \, \delta m + \int_{\delta m} \nabla_2 \cdot IV_2 r \delta m + \int_{\delta m} \nabla$$

+
$$\int_{\delta m} \frac{\partial (\omega q) \delta m}{\partial p} + \int_{\delta m} \frac{\partial (\omega r) \delta m}{\partial p}$$
 (4)

Now let us define an increment of area, $\delta \sigma$, on the vertical wall of the column, $\delta \sigma \equiv \delta l \frac{\delta p}{\rho g}$, where δl is an increment of length on the boundary on a pressure surface and ρ is density of air. Further, let C_n denote the component of IV_2 normal to the increment of area $\partial \sigma$, and defined positive outward. Then the integrals

$$\int_{\delta m} \nabla_{\!\! 2} \cdot \, IV_2 q \, \delta m$$
 and $\int_{\delta m} \nabla_{\!\! 2} \cdot IV_2 r \delta m$

transform to

$$\int_{\delta\sigma} C_{\mathbf{n}} q \rho \delta\sigma \quad \text{and} \quad \int_{\partial\rho} C_{\mathbf{n}} r \rho \sigma \tag{5}$$

through the divergence theorem of Gauss.

Let us define an increment of surface area on a pressure surface as $\delta A \equiv \delta x \delta y$.

Then the integrals

$$\int \frac{\partial (\omega \mathbf{q}) \, \delta \mathbf{m}}{\partial \mathbf{p}} \quad \text{and} \quad \int \frac{\partial (\omega \mathbf{r}) \, \delta \mathbf{m}}{\partial \mathbf{p}}$$

may be written

$$-\frac{1}{g}\int_{\delta A}\int_{\text{Surface}}^{\text{Top}}\delta(\omega q)\,\delta A \quad \text{and} \quad -\frac{1}{g}\int_{\delta A}\int_{\text{Surface}}^{\text{Top}}\delta(\omega r)\,\delta A \tag{6}$$

where the negative sign is used to accomodate the decrease of pressure from the surface to the top of the column. The transport of water vapor at the surface of the earth is the rate of evaporation assuming other processes, for example the formation of dew or frost, are neglected. The transport of water at the surface of the earth is the precipitation. It follows that the integrals (6) may be written:

$$-\frac{1}{g}\int_{\delta A} (\omega q)_{\text{Top}} \delta A - E \text{ and } -\frac{1}{g}\int_{\delta A} (\omega r)_{\text{Top}} \delta A + P$$
(7)

where E is the rate of evaporation over the area and P is the rate of precipitation over the area.

Equation (4) then may be rewritten using (5) and (7)

$$\int_{\delta m} \frac{\partial q}{\partial t} \, \delta m + \int_{\delta m} \frac{\partial r}{\partial t} \, \delta m + \int_{\delta \sigma} C_n q \rho \delta \sigma + \int_{\delta \sigma} C_n r \rho \delta \sigma - \frac{1}{g} \int_{\delta A} (\omega q)_{Top} \, \delta A$$

$$- E - \frac{1}{g} \int_{\delta A} (\omega r)_{Top} \, \delta A + P = 0 \qquad (8)$$

This equation is commonly called the atmospheric water balance equation. For notational purposes, let us denote the net flux of water through the sides and top of the volume as F_L and the change of storage of water in the volume as ΔS_L . Equation (8) then becomes:

$$E - P = \int_{\delta m} \frac{\partial q}{\partial t} \delta m + \int_{\delta \sigma} C_n q_\rho \delta_\sigma - \frac{1}{g} \int_{\delta A} (\omega q)_{Top} \delta A + F_L + \Delta S_L$$

(9)

and providing all the terms on the right-hand side of the equation can be evaluated, the exchange of water and water vapor at the earth's surface, E - P, is determined. Further, the role of the atmosphere in this exchange may be determined by observing the contributions made toward the residual by the various terms in the equation and by the contributions of individual pressure layers to these terms.

Hydrologic Balance

The same exchange of water at the earth's surface must be observed if one deals solely with the surface waters--the hydrologic balance. The hydrologic balance of the river basin may be written (Yevdjevich, 1961):

$$P - E = R_0 + \Delta W + L.$$
 (10)

Here R_0 is the runoff from the entire basin, ΔW is the change of water storage, both surface and subsurface, and L is the depletion from the river basin due to consumption within the basin and manmade diversion from the basin. Yevdjevich (1961) has determined a measure of the reconstructed runoff for the Upper Colorado where allowance was made for the consumption within the basin and manmade diversion from the basin. This reconstructed river flow is termed virgin flow, R_0^* . Then the hydrologic balance is simply:

$$P - E = R^* + \Delta W. \tag{11}$$

Because of the long-term storage in the form of snow pack in the Colorado Basin, the equivalence of P - E computed from the water balance and that from the hydrologic balance may only be tested on a seasonal and annual basis. The determination of the change in storage, ΔW , for an area of the size and topographic complexity of the Upper Colorado River Basin is most difficult. The effect on the runoff due to this carry-over of water from day to day, week to week, and even year to year, is not well understood. One method of determination of ΔW is apparent from the discussion above and that would be to evaluate the parameter P - E for a day, month, or year and subtract the runoff occurring over that time period, thus yielding ΔW (see Rasmusson, 1966). This study, however, does not include the summer months and, therefore, such an estimate of ΔW on an annual basis cannot be obtained. Riehl (1965), however, demonstrates that the annual variability in runoff from the Upper Colorado River Basin can be explained almost entirely by the variability of the winter precipitation. It is of interest, therefore, to find the relationship between the water accumulated over the winter season and the annual runoff.

Precipitation and Evaporation

Equation (9) offers a method of obtaining a measurement of evaporation providing the precipitation is known or vice-versa. The use of evaporimeters and lysimeters to estimate evaporation from water surfaces and land surfaces, respectively, has long been the main source of evaporation data. The relationship between the measurements using these devices and the actual evaporation from the natural surface is most complex and in general the instruments overestimate the actual evaporation (Sellers, 1965). This overestimation is due largely to the fact that the instrument must be isolated to some degree from the natural surface. The extension of such methods to be meaningful for large areas is most difficult.

Two methods of precipitation measurement are available: first, direct measurement using precipitation gauge data; and second, the evaluation of precipitation as a residual from the thermal balance of the atmospheric volume. Marlatt and Riehl (1963) computed the Colorado River Basin precipitation using a station network of thirteen rain gauges distributed over the basin. The station selection was based on quality and length of record. The computation

consisted of using a modified Thiessen polygon method of area weighting the precipitation data from each station. The areas were chosen so that a station represented as uniform a topographical area as possible. The daily basin precipitation, though not published in the above paper, was available to the author for this research. When referring to the basin precipitation determined by Marlatt and Riehl, the symbol $P_{\rm G}$ will be used. These data were used extensively in this work.

A test computation of the atmospheric thermal balance was attempted, but, due to instabilities in the computations and a necessary reliance upon untested assumptions, the result was discarded. The idea of isolating the contribution to the total heat budget of the volume due to the latent heat release in the precipitation process, and hence indirectly measuring the precipitation, has merit and should be pursued as the next step in the overall research program.

The following chapters will deal with the implementation of equations (9) and (11) along with the already determined basin precipitation estimate, P_{G} , with the aim to answer the problems posed in the first paragraphs of this paper.

CHAPTER III

EXPERIMENTAL DESIGN

Data

The data from the standard radiosonde network were used in the evaluation of the atmospheric water balance equation. The particular stations used in this study are shown in Figure 3. Observations over this network were taken at 12-hour intervals, 0000Z and 1200Z (0300Z and 1500Z before June, 1957). Data consisting of temperature (T), relative humidity (s), wind direction (D), and wind speed (V) along with the height of the pressure surface (z), were recorded at 50 mb increments. The temperature, pressure, and relative humidity were used to evaluate the specific humidity (q). The transformation is:

 $e = \epsilon \left[\exp \left(\frac{C_1}{T} + C_2 \right) \right]$ $q = \frac{\epsilon e}{p + e (\epsilon - 1)}$

where e is the vapor pressure, ϵ is the ratio of the molecular weights of water vapor to dry air, and C₁ and C₂ are experimentally derived constants (Holmboe, Forsythe, Gustin, 1945).

Prior to 1956, the available wind data were recorded according to a format based on the sixteen points of the compass. This format would not give the necessary resolution for the computation proposed in this paper. The data available to the author extended through April, 1963; thus the seven years, 1957 through 1963, were included in this work. This period is particularly of interest because, as already stated, over these seven years the discharge of the Upper Colorado River varied by a factor of 5, a range similar to that observed over the complete historical record.





As pointed out in the previous chapter, Marlatt and Riehl (1963) have obtained an estimate of the basin precipitation derived from 13 precipitation gauges distributed over the basin. The distribution of stations in various elevation classes is shown in Table 2 along with the percent area of the basin for the same elevation classes. There is a relative void of data from the very high elevations where the precipitation is greatest. This fact along with the well-known bias of gauge measurements due to wind effects, leads to the guess that the basin precipitation derived from gauges so distributed may be too low. The computation of basin precipitation published in the above paper covered the period 1930 to 1960 and was extended through 1963 by the author.

TABLE 2

Precipitation Gauge Network and Altitude Distribution

	> 11,000	8,000-	6,000-	
Altitude range (ft)		11,000	8,000	< 6,000
Percent of basin area	3	27	36	34
Number of Stations	0	3	8	2
Percent of Stations	0	23	62	15

Limits of the Study

As pointed out in the previous paragraphs, the experiment covered the winter seasons, 1957 through 1963, and computations of the water balance were done at 12-hour intervals.

Riehl (1965) has shown that the variation in annual basin precipitation over the Upper Colorado Basin is due almost entirely to the variation in the winter precipitation. Based on this observation, the water-balance computation was limited to the winter season, October through April. This is convenient from a computational point of view because one encounters computational problems during the summer months. The summer precipitation over the Upper Colorado Basin is usually in the form of showers and often occurs on a much smaller scale than the sampling network is capable of observing. These individual cloud systems may often be embedded in a larger disturbance; indeed, Marlatt and Riehl (1963) have shown that even in summer the large precipitation episodes cover the whole basin. Even so, the evaluation of equation (9) is tenuous under summer conditions because the radiosonde data must be assumed to be representative over distances of 300 km and over a time period of 12 hours, a scale much larger than that of the important precipitation-producing system. In winter, on the other hand, the large-scale dynamic systems causing large areas of upward motion and the associated broad areas of precipitation should be observed by the radiosonde network, and one can anticipate a successful computation.

The quantity of water vapor in the atmosphere decreases rapidly with height so that the depth of the atmospheric volume used in this computation may be limited. For example, Figure 4 shows the average vertical distribution of specific humidity over Grand Junction, Colorado, during March, 1961. The radiosonde device fails to measure the humidity if the water vapor content becomes very small and in this event a statistically derived value is entered into the data; this procedure is used approximately half the time during the winter above 500 mb in the Grand Junction data. Because of the spurious errors caused by this procedure and because of the relatively small amounts of water vapor above 500 mb, the assumption was made that at and above 475 mb the water vapor is negligible (q = 0). The assumed profile is also shown in Figure 4. The above assumption amounts to a discard of about 5 percent of the total water vapor content. The limits to the study may be summarized as follows: The seven winters, 1957 through 1963, were studied; the computation was performed at 12-hour intervals over these seven winters; the atmospheric column extended from the surface to 475 mb over the area of the Upper Colorado Basin.

Finite Difference Scheme

The radiosonde stations, Figure 3, are distributed over the map in a random fastion. To evaluate the integrals in equation (9), the data were interpolated to a grid on the boundary of the basin. The interpolation from the data points to the grid points was done with an objective analysis scheme based on the fitting of quadratic surfaces to each variable on each pressure surface. The particulars of the scheme are given in Appendix A. Figure 5 shows the nine-point boundary grid chosen for the analysis. The average elevations of the earth's surface (Z_s) along with the length of the line increments ($\Delta \ell$) centered on the grid points are listed in Table 3. A tenth grid point was located interior to the basin and coincides with the locatior of the Grand Junction radiosonde station.

Point	Surface Height Z _S (m)	Length of Line Increment (km)
1	2620	260
2	2570	250
3	2970	260
4	2370	260
5	2070	250
6	1920	250
7	2100	260
8	2360	260
9	2400	260

TABLE 3 Surface Height and Boundary Length for Each Point of the Boundary Grid



Figure 4. Average vertical profile of specific humidity at Grand Junction, Colorado, for March, 1961. The assumed profile is given by the dashed line.





Because of the mountainous terrain, one may not assume that the earth-atmosphere boundary is at a uniform height. The total area, A, or boundary length l may vary from level to level depending upon how much of the area or boundary is in the atmosphere and how much is interrupted by the topography of the earth's surface. To obtain average values of quantities on a pressure surface over the area and on the boundary, each point was allotted an element of area ΔA (Figure 6) and an element of boundary length Δl (Figure 5). The superscript notation to be followed for the remainder of the discussion will be:

= area averaged quantity
 = deviation from the area average
 > = boundary averaged quantity
 * = deviation from the boundary average

The area average of any quantity, ξ , may be written

$$\overline{\xi} = \frac{1}{A_j} \sum_{i=1}^{n} \xi_{ij} \Delta A_{ij}$$
(12)

where $A_j = \sum_{i=1}^{n} \Delta A_{ij}$. The subscript i refers to data or operations on a particular pressure surface and the subscript j indicates operations on different pressure surfaces. Similarly, the boundary average on a pressure surface of any quantity, ξ , may be written

$$\widehat{\xi} = \frac{1}{\ell_j} \sum_{i=1}^m \xi_{ij} \Delta \ell_{ij}$$
(13)

where $\ell_j = \sum_{i=1}^m \Delta \ell_{ij}$. It follows from (12) and (13) that

$$\xi_{ij} = \overline{\xi_j} + \xi_{ij} ; \quad \xi_{ij} = \widehat{\xi_j} + \xi_{ij}^*$$
(14)

and that

$$\xi' = 0 ; \quad \xi = 0$$
 (15)

The primed and starred items are termed area and boundary "eddy" terms, respectively.



Figure 6. A rea increments used to obtain the area weighted averages.

Because of the uneven terrain, the lowest layer may not be the standard 50 mb increment. At some points around the boundary on a pressure surface the layer may be totally, partially, or not at all above the earth's surface. This topographic variation was incorporated in the computation by employing a weighting factor ψ_{ij} which normalized the data to 50 mb layer values. The weighting factor may be expressed: $\psi_{ij} = \frac{\Delta P_{ij}}{\Delta P}$ (16) where $\Delta P = 50$ mb.

The normalized quantities are noted by a tilde

$$\widetilde{\xi}_{ij} = \xi_{ij} \psi_{ij} \tag{17}$$

Figure 7 illustrates the evaluation of the weight factors The scheme is based on the approximation of a linear relationship between pressure and height which, while not exact, is a good first approximation over small pressure intervals (e.g., 50 mb).

Following the notation as shown in Figure 7, the weighting factors were evaluated from the height profile at each point as follows:

$$\begin{split} \psi_{ij} &= \frac{\Delta P_{ij}}{\Delta P} = \frac{H_{j+1/2} - Z_s}{H_{j+1/2} - H_{j-1/2}} \text{ where } H_{j-1/2} \leq Z_s \leq H_{j+1/2} \\ \psi_{ij} &= 0 \qquad \text{where } Z_s > H_j + 1/2 \\ \psi_{ij} &= 1 \qquad \text{where } Z_s \leq H_j - 1/2 \end{split}$$

Here $H_{j+1/2} = \frac{Z_{j+1} + Z_j}{2}$ and $H_{j-1/2} = \frac{Z_j + Z_{j-1}}{2}$

Table 4 gives a numerical example of the computation of the $\psi_{\mathbf{i}\mathbf{j}}$ values.

The atmospheric water balance equation (9) written in finite difference form and incorporation the averaging notation (12), (13),



Figure 7. Scheme for obtaining weighting factors. $\psi_{\rm ij}$.
and (14) along with the weight factor notation (17) is:

$$P - E = -\frac{\widehat{\Delta P}}{g} \underbrace{\frac{\Delta}{\Delta t}}_{j=1} \underbrace{\overset{7}{\overline{q_{j}}}}_{j=1} \underbrace{\widetilde{q_{j}}}_{j=1} A_{j} - \underbrace{\widehat{\Delta P}}_{g} \underbrace{\overset{7}{\Sigma}}_{j=1} (\widehat{C_{n} q})_{j} \ell_{j} - \underbrace{\widehat{\Delta P}}_{g} \underbrace{\overset{7}{\Sigma}}_{j=1} (\widehat{C_{n} q})_{j} \ell_{j} + [(\underbrace{\widetilde{\omega}}_{q} q)_{j} = 7] \underbrace{A}_{g} + [(\underbrace{\widetilde{\omega}}_{q} q)_{j} = 7] \underbrace{A}_{g} - F_{L} - \Delta S_{L}$$
(18)

Here the vertical summation indices $j = 1, 2, 3 \dots 7$ correspond to pressure levels $p = 800, 750, 700 \dots 500$ mb, respectively.

Simplification of the Water Balance Equation

The standard meteorological sampling network does not measure directly the amount of liquid water or ice in the atmospheric column and, thus, the terms ΔS_{I} and F_{L} of equation (18) are not easily evaluated. In most research using the atmospheric water balance equation, these terms are justifiably neglected since they are of second order in magnitude when compared to the water vapor terms (Palmen, 1967). It is not readily apparent that one should neglect these terms when dealing with mountainous areas, however, because of the selective cloud patterns resulting from the effect of topography on the air flow. Two general types of clouds exist over the Colorado River Basin in winter; the large masses of stratiform cloud associated with a large scale synoptic disturbance, and standing mountain wave clouds located predominantly over and to the east of the high mountain range forming the eastern boundary of the basin. It is necessary that the order of magnitude of the terms ΔS_L and \mathbb{F}_L for these two types of cloud systems be evaluated. The following order of magnitude argument is designed to provide extreme examples of the possible magnitudes of the liquid water terms.

TABLE 4

Example of the Weight Factor Computation. Data is for Grid Point 8, (i = 8), 13 March, 1961, 1200 Z. Surface Height Z_S = 2359 m.

j Index	Pressure Level (mb)	Pressure Height Z _{i,j} (m)	Layer Mid-Height $H_{i j \pm \frac{1}{2}} = \frac{H_{i, j \pm H_{i, j \pm 1}}}{2}$ (m)	Compare $H_{i, j \pm \frac{1}{2}}$ with Z_{s}	$\frac{H_{j+\frac{1}{2}} - Z_{s}}{H_{j+\frac{1}{2}} - H_{j-\frac{1}{2}}}$	ψ _{i,j}
1	800	2021	2277	Z _S >H _{i+1/2}		0
2	750	2532	2809	$H_{j-1/2} < Z_s <$	450/532	. 85
3	700	3078	3350	$H_{j+1/2}$		1
4	650	3664	3950	$Z_{s} < H_{j-1/2}$		1
5	600	4287	4616	n		1
6	550	4945	5309	- in-		1
7	500	5674		T.		

First, let us consider a large-scale cloud system covering the entire basin. If one assumes a cloud 500 meters thick covering the basin and having a liquid water density of $.1 \text{ gm/m}^3$, the water held in this cloud has an equivalent depth over the basin of 0.05 cm. This is an order of magnitude less than the precipitable water vapor content over the basin which varies from a monthly mean of 0.6 cm during January to over 2.0 cm during August (Reitan, 1960). If one further assumes that the processes resulting in advection and local change are not different for vapor and liquid, then the terms ΔS_L and F_1 may be justifiably neglected for this cloud system.

The problem of the standing mountain-wave cloud is not as simple to formulate. Let us assume a cloud of density $.1 \text{ gm/m}^3$ extending 800 km along the eastern border of the basin and having a vertical extent of 2000 meters. Further, let us assume a wind of 30 mps invariant with height and normal to the boundary. Such a system would advect out of the basin per day the equivalent of 0.1 cm of water distributed over the basin.

If one neglects the liquid water terms this omission would be counted as precipitation in the balance equation because the water entered the basin in the vapor state and was advected out of the basin in the liquid state. Such a process imposes a systematic error on the computation with the order of magnitude being as high as .1 cm per day, a sizeable contribution if accumulated over a winter season. This apparent problem is offset, however, by the computational procedure. The mountain-wave cloud forms on the upwind side of the range and evaporates on the downwind side of the range. The boundary data used in the computation are the result of a surface fitting technique described earlier in the text and uses data from both sides of the range with most of the data obtained from locations well away from the mountain wave cloud and where the cloud water is again in the vapor state and thus measured. Only that portion of the water that is transported through the 500 mb surface in the cloud and which does not return as vapor to levels below 500 mb in the lee of the mountains is not measured and, thus, is still erroneously counted as precipitation. In summation, then, the neglect of the liquid water terms in equation (18) causes only errors of second order in magnitude. Systematic errors of something less than .1 cm per day of water distributed over the basin are possible through the mechanism of the mountain wave cloud.

The vertical transport terms, $(\widetilde{\omega} q)_{j=7}$ and $(\widetilde{\omega} q')_{j=7}$ are neglected. One does not measure the eddy vertical motion w' on the scale where this term is perhaps most important, the scale of individual clouds. This problem was discussed previously and is precisely why the study is restricted to the winter season where the term is perhaps less important than during the summer season. The inability to evaluate this term is a severe restriction for this study.

The expression for the atmospheric water balance after taking into account the simplifications listed above becomes:

$$P-E = \frac{\Delta p}{g} \left[\frac{\Delta}{\Delta t} \sum_{j=1}^{7} \overline{\widetilde{q}_{j}} A_{j} + \sum_{j=1}^{7} (\widehat{\widetilde{C}_{n} q})_{j} \ell_{j} + \sum_{j=1}^{7} (\widetilde{\widetilde{C}_{n} q})_{j} \ell_{j} \right]$$
(19)

and is the expression evaluated to determine P-E as a residual.

Details of the Water Balance Computation

The C_n field: The problem of obtaining accurate measures of mass divergence and hence vertical motion has long been a major problem in any meteorological analysis. Since the computation performed here is dependent to a large degree upon the normal wind component, C_n , obtained from the objective analysis scheme, and, therefore, the divergence, it is valuable to test this particular parameter. One method of evaluation is to compute the vertical motion at the top of the atmospheric column (475 mb) using the C_n values from the analysis and compare this vertical motion with a corresponding vertical motion obtained independently using another method. The independent measure used here was the vertical motion at 500 mb computed from the vorticity equation and published by the U.S. Weather Bureau in the form of analyzed maps. It was assumed that the mean vertical motion over the top surface at 475 mb and 500 mb were not systematically different.

The vertical motion computation is based on the continuity equation (2), integrated over the atmospheric column extending from the surface to 475 mb. Assuming that $\omega = 0$ at the earth's surface and using the notation outlined above, one obtains

$$\overline{\omega_7} = -\frac{\Delta p}{A} \sum_{j=1}^7 \overline{\widetilde{C}}_{nj} \ell_j$$
(20)

The values were converted to vertical velocity (w) using the relationship

$$-\frac{\omega_7}{\rho_7g} = w$$

where $\overline{\rho}_7$ is the average density at 475 mb. The comparison of the two fields is shown in Figure 8. The data were obtained from a random selection of individual 12-hour analyses and computations during the water year, 1961. The Weather Bureau product shows less dispersion, in part due to the smoothing caused by the visual interpolation from analyzed charts, and in part due to the fact that the vertical motions computed using equation (20) above build in the influence of topography to some degree. The correlation between the two measures is good, r = .8. This analysis, while not conclusive, shows that the C_n values are meaningful and not wholly masked by computational error.



Figure 8. Vertical motion at 475 mb computed from the C_n data plotted against vertical motion at 500 mb putlished by the U.S. Weather Bureau.

The procedure for evaluating the daily atmospheric water balance: It is of interest to inspect the contributions of the various vertical elements to the individual terms of equation (19) and further to inspect the total vertically integrated terms as they contribute to the daily residual P-E. The terms of the water balance equation are computed every 12 hours and it is desirable to define a "day" as the 24-hour time increment which corresponds to the 24 hours over which the precipitation measurement, P_G , is taken. The 13 precipitation gauge measurements, however, are not taken at a uniform time; the observation times varied from 2300Z to 0700Z the next day. In order to best fit the data, the daily P_G values were defined as representative of the precipitation falling over a 24-hour period centered on 1800Z of the day of record. Thus a "local" time, subscript L, based upon this period can be defined as:

$$t_{L} = t_{Z} - 6$$

where t is the time in hours.

All the daily evaluations of the water balance equation were based on this "local" time definition. The terms of the equation were summed for a daily value as follows: Let

$$\sum_{j=1}^{7} \overline{\widetilde{q}} A_j = W$$

then

$$\frac{\Delta}{\Delta t} W_{\text{DAY 1, L}} = \frac{W_{\text{DAY 2, 12Z}} + W_{\text{DAY 2, 00Z}} - W_{\text{DAY 1, 12Z}} - W_{\text{DAY 1, 10Z}}}{2}$$

where t = 24 hours. Let

$$\sum_{j=1}^{7} (\widehat{C}_{n} q)_{j} + \sum_{j=1}^{5} (\widehat{C}_{n} q^{*})_{j} = \chi$$

then

$$DAY 1, L = \chi DAY 1, 12Z^{+\chi} DAY 2,00Z$$

In these expressions, time is counted according to the "local" time definition on the left-hand side and according to Z time on the right-hand side of the equal sign. This somewhat arbitrary scheme does not force absolute equivalence between periods of the precipitation data and of the water balance computation, but it is the only feasible method. Not much credence was placed on the daily values per se, but rather on periods lasting several days, then this problem of corresponding time increments is minimized.

Detailed Analysis of the Water Balance Computation for One Month, October, 1960

Because of the large number of individual computations, it is not useful to inspect every 12-hour computation over the seven winters in order to evaluate the importance of the various vertical elements to each term of equation (19) and to demonstrate in general the contribution of each of the terms to the daily residual P-E. For this purpose the detailed analysis of one month will be used to demonstrate the pertinent points. The month chosen for this analysis was October, 1960. The month was selected because it contains two distinctly different periods in its daily course of precipitation (Figure 9). First, the large precipitation episode from the 8th to the 18th was caused by two massive, slow-moving cyclones passing over the western United States. Figure 10 is the 500 mb map for 10 October, 1960, 0000Z (9 October 1800L) and clearly shows the type of atmospheric disturbance associated with the very large precipitation occurrences over the Urper Colorado River Basin. Secondly, the relatively dry period following the precipitation episode was associated with a dominant anticyclonic circulation just upstream from the basin. A representative 500 mb map for this period is shown in Figure 11.

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Figure 9. The daily course of $\,{\rm P}_G$ for October, 1960.







Figure 11. The 500 mb map for 26 October, 1960, 0000Z. Contours (solid lines) are in 100's of feet msl. Isotherms (dashed lines) are in degrees centigrade. The local change with time of water vapor in the column: Figure 12 is the daily vertical-time section of the local change in water vapor over the Upper Colorado River Basin during October, 1960. The section shows continuity in both space and time. with the largest contribution to this term appearing just prior to the large storm. The rest of the section appears quite flat. The magnitude of the contributions are a maximum in the lower and middle layers due to the fact that the water vapor content decreases so rapidly with height. The signs and magnitudes of the isolines indicate their contribution to the residual P-E. The large negative values, therefore, indicate an increase with time of water vapor over the basin prior to the large disturbance.

Divergence of water vapor flux terms: Figure 13 is the daily vertical-time section of the divergence of water vapor flux due to the mean wind

 $- \frac{\Delta p}{g} (\widehat{\widetilde{c}_n q})_j \ell_j$

for October, 1960. The signs and magnitudes of the isolines indicate the contribution from this term to the residual P-E. A positive sign, therefore, indicates a net inflow of water vapor due to this term. Good continuity is obtained both in space and time and a definite decreasing contribution with height. The large contributions by this term are found during the precipitation episode and again in the dry period.

Figure 14 is the vertical-time section of the eddy divergence of water vapor flux

 $-\frac{\Delta p}{g} (\widetilde{C}_n^* q^*)_j \ell_j$ for October, 1960. The eddy term exhibits a much flatter pattern over the entire section, but also has continuity in space and time as do the other terms. Strong contributions during the precipitation episode are not as evident as for the mean divergence term.

Figure 12. The daily vertical-time section of the local change of water vapor over the upper Colorado River Basin during October, 1960. Units are cm of water per day distributed evenly over the basin. Negative values show an increase with time of water vapor in the atmospheric volume over the basin.



Figure 13. The daily vertical-time section of the mean divergence of water vapor flux during October, 1960. Units are cm of water per day distributed evenly over the basin. Positive values show a net import of water into the atmospheric volume over the basin.

550 600

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Figure 14. The daily vertical-time section of the eddy divergence of water vapor flux during October, 1960. Units are cm of water per day distributed evenly over the basin. Positive values show a net import of water into the atmospheric volume over the basin. Vertically integrated terms of the water balance equation: Figure 15 shows the daily course of the vertically integrated terms of the water balance along with the daily residual P-E for this one month. Also shown is the daily course of P_G and P-E is evident. Days with net evaporation, negative P-E, over the Upper Colorado River Basin are observed.

Summary of the detailed analysis: In general this detailed analysis of one month of the atmospheric water balance demonstrates that the computation exhibits both space and time continuity for all terms of the water balance equation. Each of the terms can have the same order of magnitude and, in general, the major contributions to the terms come from the lower layers of the atmospheric volume. The large contributions from the mean divergence of water vapor flux demonstrate that the ageostrophic portion of the wind field is indeed important in the water balance computation and cannot be neglected for computations over this area size as often has been done in similar computations over larger areas (Morrissey, 1964; Benton and Estoque, 1954). The good agreement in daily trend between the residual, P-E, and the basin precipitation estimate, $P_{\rm G}$, along with the space and time continuity of the vertical elements of each term, provides for confidence in the computation.

Sources of Error in the Atmospheric Water Balance and Basin Precipitation Computations

Several sources of computational and sampling error have been mentioned in the preceding sections of this paper. This section will serve the purpose of listing these and other error sources and, where possible give estimates of the possible magnitude of the errors. Some of the numerical values have been obtained from previously published papers and because of the variety of experiments from which these estimates are drawn, perfect correspondence cannot be expected.



Figure 15. Top: The vertically integrated values of the three terms in the atmospheric water balance. For each day the three bars represent the local change (left), mean divergence of flux (middle) and the eddy divergence of flux (right) terms, respectively, a positive value indicates a positive contribution to the residual (P-E).

Bottom: The daily course of P-E computed from the atmospheric water balance (solid line). The daily course of ${\rm P}_{\rm G}$ (dashed line).

Errors in the atmospheric water balance computation: Hutchings (1957) did a thorough error analysis of an atmospheric water balance computation and concluded that the primary source of error is due to the 12-hour sampling interval. This sampling error is random in nature and may be suppressed through summation of consecutive daily values. Errors arising from instrument deficiencies including instrumental lags are, according to Hutchings, small compared to the sampling error. His analysis is based upon a water balance computation done over southern England during summer (June-August). The area was approximately one-third the area of the Upper Colorado Basin and the computation was done using only four radiosonde stations. The results published in the above paper showed that the standard error due to all sources in the divergence of moisture flux computation amounted to 50 percent of the water distributed over the area for the three-month period. Rasmusson (1966) pointed out that one can expect the magnitude of the error to decrease as one increases the size of the area, increases the number and density of radiosonde stations, and increases the period of summation. No precise estimate is available for an area the size of the Colorado Basin and for an analysis incorporating the smoothing benefit of an objective analysis using many more radiosonde stations. Rasmusson (1966) further isolated a source of systematic error due to the diurnal variation in the wind, particularly in the lower layers of the atmosphere. The error from this source arises from the fact that the procedure of sampling the atmosphere only twice a day does not define the diurnal variation. The error due to this source is predominantly a summer phenomenon. From the data presented in the above paper, the magnitude of this error over the Colorado Basin is less than 0.01 cm per day during the winter.

The neglect of the liquid water terms in the balance equation has been discussed in detail in preceding sections of this paper and

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amounts to an error of negligible magnitude except perhaps under the condition of a massive standing wave cloud over the Continental Divide. Under such conditions, errors of 0.10 per day are possible.

In summary, then, the sampling procedure imposes the greatest source of error on the water balance computation. This error diminishes as one sums over an increasing period of time. Systematic errors of appreciable size can be obtained due to the diurnal variation of the wind and also due to orographically induced cloud configurations.

Errors in the basin precipitation estimate: As pointed out in the Introduction and reiterated in the preceding chapter, the precipitation estimate derived from gauge measurements is biased toward the low side; this is particularly true in the case of snow. The effect on the snow catchment is primarily related to wind speed and is most serious for the standard unshielded precipitation gauge (Weiss and Wilson, 1957). With a wind of 8 mps the catchment of a standard gauge is only about 50 percent. Considerable improvement is observed if one uses shielded gauges. Of the 14 gauges used to determine P_{G} , only one was of the shielded variety and, thus, the underestimate of basin precipitation can be extreme due to this measurement problem.

The problem of obtaining a meaningful network of gauges for a large mountainous area is also of concern. The gauges are biased toward the low elevations and their density is very low. The net result of these two aspects of measuring precipitation over mountainous regions leads to a further underestimation of the areal precipitation (LaRue and Younkin, 1963).

In summary, then, the errors inherent in the measurement of precipitation, particularly snow, are systematic and lead to an underestimate of the basin precipitation. The errors on individual days vary and cannot be easily corrected because the effect is largely due to local wind conditions at each gauging site.

CHAPTER IV THE ATMOSPHERIC WATER BALANCE

The summarized results of the complete seven winter experiment will be presented in the following sections. The daily, monthly, and seasonal results will be treated separately. In addition, a "natural period" analysis will be presented; the natural periods are delineated by periods showing homogeneity in the parameter P-E over consecutive days and thus are more physically meaningful than summations over arbitrary chronological periods.

The Daily Atmospheric Water Balance

Not much credence can be placed on the daily values of the parameter P-E computed as a residual of the atmospheric water balance computation due to the various sources of error enumerated in the preceding chapter. The daily values of the parameter P-E and the daily values of the precipitation estimate P_G are given in Table 5. In addition, the daily time series of these two parameters and their three-day running averages are plotted in Figures 16a through 16g. From these diagrams it is observed that much of the apparent computational instability in the daily P-E regime is smoothed out in the three-day running average series. Further, from a visual inspection of the time series, it is evident that the daily course of P_G is clearly reflected in the daily course of P-E. The lag that is apparent on many days between the two parameters $\ensuremath{\operatorname{P-E}}$ and $\ensuremath{\operatorname{P_G}}$ can be attributed to the different sampling times of these parameters. In general, days and periods with large basin precipitation values show good agreement between the two parameters, and periods with no precipitation correspond to periods with negative values of P-E, the case where evaporation dominates. Days and periods with

IABLE 5. DAILY VALUES UF P-E AND P FOR WATER YEAR 1957. (UNITS--CENTIMETERS OF WATER DISTRIBUTED OVER THE BASIN)

	14													
	001	OBER	NUVEN	MBER	DECEN	ABER	JANL	JAHY	FEARL	JAHY	MM	ARCH	A	HL
AY AY	9-6	9 d	p-F	a.	B-E	a	д- Б-	đ	д-д	ر ۲	u e d	٩	4-1	a
-	.73	.13	92.	.16	18	• 00 •	• 22	.00.	. 4 1	.10	. 14	•13	80.	14.
N	• 54	00.	15	.10	- 23	•00	• 28	.08	.80	• 00	.50	•08	· U2	4 4 .
m	55	• 00	.20	• 0 8	03	.00	.71	• 02	.10	• 02	.22	. 43	-05	•18
4	.61	• 00	. 4 U	¢0•	.32	• 00	• 25	94.	.30	• 00	-,14	• 05	20	¢0.*
s	.45	• 0.0	.02	•58	.39	• 30	12.	64.	•16	• 0.0	.31	.16	•28	-05
9	15	• 0.0	.22	•18	.30	.51	16	e1.	.13	• 0.0	05	.15	28.	•16
2	. 73	• 00	23	•10	84.	84.	22	• 05	.50	• 00	14	•13	10.	.13
r	.37	- 02	.01	• 0 0	.18	.13	.83	1.15	• 15	• 33	37	.10	.17	• 0.0
0	.38	*08	02	• 0 0	.41	• 00	• 0.8	• 66	15	80.	.59	• 0.0	-01 -	• 0 0
10	.89	.00	.01	•00	.83	-02	•13	• 0.8	.33	• 02	.38	.43	15	• 0.0
11	11.	00.	16	• 0.0	81.	.45	•20	-02	10	• 0.0	.24	•13	17	. 20
12	.88	•13	.11	• 00	•15	04.	.12	•15	05	• 00	.60	•12	35	.15
13	*28	.15	•10	• 0 2	1.01	.15	• 85	1.07	+1+	• 0.0	.21	• 9 4	1.5	•10
14	.23	• 0.0	.01	• 28	-,06	.70	01	•58	c0.	• 00	14	• 00	.10	• 00
15	• 00	• 0 0	00	• 0.0	.26	• 00	10	•18	03	• 00	.09	00.	25	• 08
16	.40	• 00	12*	80°	.10	.15	18	• 08	.22	• 00	54.	• 0.0	• 05	90.
11	01	00.	.28	.02	90°	- 05	02	.00	• 9 •	• 0H	10	• 23	• 0 •	•15
18	- 42	• 00	•16	.13	12.	• 00	•0•	.00	.26	•08	.10	.00	• 0.0	¢0•
19	03	.02	•29	• 15	+2	•00	10.	• 00	.25	•10	.08	• 00	•17	÷1.
20	ćð.	.00	23	• 0 B	.10	• 00	07	• 0 A	.32	•08	.06	• 02	25	• 0.0
51	12.	• 00	• 02	• 00	. I &	• 0.0	•31	84.	04.	• 38	.58	• 05	32	.00
22	11	00.	.30	• 00	E0	.10	.21	· 02	.10	•10	.10	.48	• 25	94.
63	1.28	.30	• 00	• 00	10.	= 02	•37	• 08	1.01	•18	20	• 00	•29	1.10
42	•30	66 *	03	• 00	02	• 02	• 00	.41	.10	• 38	20	• 0.0	• 28	•10
52	.61	.13	.06	•00	£0°	• 00	•28	84.	÷0*	.30	*2A	• 02	•03	.18
56	.48	. 23	• 00	• 00	.12	•00	.87	• 96	.08	• 05	34	•15	•13	• 02
27	.40	• 05	05	•00	17	• 00	17.	•66	.30	• 05	.05	• 00	17	• 05
58	.61	• 36	02	• 0.0	e0.	• 00	[4 •	•61	15	•13	E0.	• 00	•29	90°
54	.55	.53	16	•00	£1.	• 00	.18	.10	• 00	• 00	.26	• 00	12.	• 66
30	.70	•13	.20	• 00	.12	• 00	•17	•10	• 00	•00	.42	• 08	• 25	.13
31	44	-10	00-	-00	0.0	00-	10	E 1 -	-00	-00	10	.13	-00	• 0 0

TABLE 5. DAILY VALUES OF P-E AND P G FOR WATER YEAR 1958. (UNITS--CENTIMETERS OF WATER DISTRIBUTED OVER THE BASIN)

	OCTO	DHEH	NOVE	MBER	DECEN	MBER	JANI	JARY	FEBRI	JAHY	MA	ARCH	AH	RIL
DAY	P-E	PG	P-L	PG	P-E	PG	P-E	PG	P-E	PG	P-E	PG	P=E	PG
1	20	.00	.00	.+0	12	.00	.00	.00	09	.02	02	50.	.16	.10
2	.05	.02	.97	.20	U3	.02	06	.00	02	.00	06	.02	.20	.36
3	.35	.23	.71	.84	.02	.00	.05	.00	.20	.00	.20	.00	.27	.08
4	23	.28	. 52	. 41	. 02	.00	37	.00	.57	.20	.19	.02	.43	.71
5	11	.08	.45	. 43	1.11	.30	29	.00	.22.	.33	07	.00	.20	.33
6	42	.08	.30	.33	.22.	.48	.00	.00	.08	.15	.46	.02	09	.00
1	03	.02	07	.02	14	.41	.20	.00	10	.00	.01	.10	09	.00
н	44	.00	. 02	.02	30	.05	11	.00	05	.10	.20	.08	.00	.38
9	.05	.00	.07	.00	.08	.00	07	.00	12	.33	.27	. 41	60	.08
10	.20	.00	12	.00	11	.00	.17	.02	.27	.33	17	.00	.07	.02
11	.70	.10	.33	. 0 B	.00	.00	.06	.10	16	+08	.27	.10	30	.00
12	.05	.23	.05	.05	21	.00	11	.00	. 49	.05	.20	.30	.16	.10
13	.77	.99	.23	.15	13	.00	.30	.02	.51	.66	.00	.10	10	• 0 0
14	65	. 94	.75	.43	.63	.00	16	• 0 2	.07	.02	22	.13	• 0 0	•00
15	05	.10	07	.45	.32	.08	08	.00	.02	.25	.35	.20	08	.00
16	.40	.02	.22.	.20	.70	.61	23	.00	50	.53	.90	.76	.06	.00
17	01	.10	10	.18	.36	.28	.09	.00	55	.00	. 40	.13	28	.00
18	.67	.08	-,13	.02	.13	.10	.54	.10	.11	.00	16	.25	.28	.02
19	.31	.40	.07	.38	.10	.28	.12	.13	.02	.00	12	.00	08	.02
20	.52	.94	15	.08	17	.05	09	.05	.08	.00	11	.00	.07	.00
21	.55	.28	03	.02	.43	.02	01	.00	06	.00	.20	.10	26	.00
22	.33	.18	.10	.00	.03	.02	15	.00	.11	.00	.30	.33	.88	.15
23	.20	.00	31	.00	.12	50.	.20	.00	.65	.10	.09	.08	.30	.30
24	15	.10	.13	.00	.04	.18	40	.28	06	.05	.06	.20	• 31	.05
25	20	.00	.03	.00	.48	.10	.25	.20	1.00	.69	.24	.30	25	.05
26	.00	.00	.23	.00	20	.02	.00	.10	.01	.18	17	.53	21	.02
27	.93	.08	25	.05	.38	.13	.10	.33	01	.05	.14	.00	.07	.00
28	.17	.13	.00	.05	.09	.15	.11	.08	06	.08	10	.36	05	.10
29	30	.02	20	.02	.08	.18	.06	.15	.00	.00	.05	.08	.10	.02
30	15	.00	.03	.00	.09	.25	.65	.18	.00	.00	.25	.02	•00	.00
31	A 14	- 110	0.0	- 011	0.0	0.0		1 3	0.0	0.0	27	. 23	- 0.0	- 0.0

HABLE 5. UAILY VALUES OF M-E AND P. FOR WATEN YEAR 1959. (UNITS--CENTIMETERS OF WATEN DISTRIBUTED UVEN THE BASIND

	100	OBER	NUVE	MUER	DECEN	NBER	JANC	JAHY	FEBAL	IAHY	W	NRCH	A	PRIL
740	3-1	2	đ	a ^c	J-d	4	and a	a	Per	ď	u d	51	b-d	BG
-	.54	.00.	10	.00	12	.00.	-52	•02	-02	.08	10	•00	. 25	• 00
v	-19	-02	04	•00•	.23	•00	.10	•08	•28	• 23	37	•00	•20	• 00
Υ	10.	00.	.10	•00	12	.00	•20	<0.	П.	•05	.20	•05	12.	• 00
÷	.18	• 0.0	.14	.00	.14	•02	E0	•00	.31	.40	+0	-05	-•05	• 00
ŝ	. 72	• 0.0	05	.20	50	.00	+0°-	• 00	06	E1.	16	• 00	• 05	• 0.0
S	• 34	-02	+1	•20	. *5	• 0.2	.45	•08	· 35	• 0 0	14	.15	10.	• 0 0
1	10.	• 0.0	.01	•1e	.00	<0.	12	•23	.09	- 02	.30	• 05	.75	• 56
r	EE.	•00	e1.	·13	ET.	.10	.18	•00	.80	.20	.22	.13	•16	*2A
3	14.	• 00	.14	•00	10	•23	.17	•00	•01	11.	11.	• 00	10	·25
10	16.	•00	-25	•1H	14	-15	•I.	• 0.0	. 25	.10	E0	•15	+1+	• 00
11	.18	.00	c1.	.10	17	*3H	13	.00	68.	-02	.00	• 05	12	\$0*
14	.52	00.	¢¢.	.28	00	e1.	22	•00	÷5.	.20	CO.	• 0 0	11	•00
13	·15	• 0.0	.20	c0.	13	<0*	•18	• 00	09	.20	.65	•10	60°	• 0 0
14	25	• 0.0	.60	64.	S0.	• 00	.18	• 02	.17	• 00	10.	•20	EE.	• 00
15	¢0.	• 0.0	•23	•7B	00	• 00	* °] 4	•13	16.	• 00	25	• 00	.13	•10
1 tr	10	• 0.0	.31	*38	+2	•00	•00	•20	16.	¢۶.	.14	• 0.0	02	•08
11	-11	• 00	• 00	.10	· 40	• 0.0	*I*	• 02	• 35	•56	.10	•00	•29	• 00
18	.15	•00	.10	• 0.0	EL.	•00	•00	•00	60°	.15	12.	\$0.*	.20	•23
51	.82	• 02	•00	•00	20	• 00	• 00	•13	11	•10	.25	•25	•15	•30
50	+ * * -	.18	.02	.00	\$2.	• 00	•24	80°	.28	• 05	+0*-	• 02	14	• 05
12	40 *	• 0.0	+0+	• 0.0	12	• 00	02	.00	.33	• 02	n0 *	• 0.0	10	• 05
25	07	• 00	02	•00	64.	• 00	-15	s0.	.10	•76	0B	• 00	.20	• 00
63	10.	00.	60*	•00	.20	¢2.•	•11	•00	12	•15	60*	• 00	30	• 00
47	.18	• 00	e1.	• 00	+2 ·	20.0	-07	• 0.0	-11	• 05	.0g	• 02	.27	•00
52	.50	14.	87°	-05	.38	20.	·33	•13	+0-	• 05	06	•15	- 55	• 00
56	21	•18	62.	×0.4	.20	.10	02	·13	.16	• 05	11	• 05	•09	64.
12	+°08	•00	. 45	• 33	15	• 02	•2*	•00	""II	•20	.40	• 05	16.	.61
Ť	*0*	80.	10	•00	.32	·15	30	•18	60*-	• 02	01	.13	.20	.00
57	¢1.	.20	01	•00	12	-05	*16	•25	•00	• 00	\$2.	•18	.23	•00
30	18	c0.	• 00	• 0.0	• 05	•05	•11	• 00	•00	•00	.50	•10	•20	00.
In	02	• 00	.00	00.	.28	c0.	06	-05	-00	.00	20-	-36	•00	- 0.0

IABLE 5. DAILY VALUES UP P-E AND P FOR WATER YEAR 1960. (UNITS--CENTIMETERS OF WATER DISTRIBUTED UVEN THE BASIND

WATEN YEAR 1960

	0010	DAER	NUVE	MBER	DECEN	ABER	JANL	JARY	FEBAL	JARY	MA	NHCH	AF	HIL
VAU	9-1 1	J	b-F	n	ш. - д	2	ш- -	9	3-d	9.9	ų. P	£	ш.,	8
-	54.	• 69.	14	• 05	.10	00.	.15	.28	52.	• 00	.30	.41	HU.	BI.
ν	.16	.34	.63	.20	.15	• 00	• 00	• 05	.80	.69	.11	•23	20	S0.
m	16	.10	46.	.20	.10	.00	.13	• 0.0	+0.1	.18	.17	.13	00.	.00
*	25	.00	čô.	• •	21	.00	05	• 00	16	• 00	14.	.20	12	• 00
5	.01	• 0 0	04	.00	10	• 00	06	.00	.12	.05	.23	.18	.30	• 0 0
r	*2*	• 0.0	60.	• 0 0	61.	.00	61.	•13	.07	.02	.00	• 08	64.	• 0 0
2	.40	64.	¢0.	• 00	.18	.00	•2•	•13	.50	.13	· 65	R0.	16	• 00
τ	34	• 0 0	15	.00	02	• 0.0	50	• 02	.86	.30	.24	.23	16	• 05
7	.81	•13	. 21	• 0 0	00.	• 0 0	.18	.00	69.	• 89	.07	• 05	•22	.00
1.0	E0	PO.	• 0.0	• 00	60.	.10	• • 5	• 00	.13	.30	26	• 0.0	•06	.00
11	. 4 5	~ 0 ~	11	.00	20.	-02	.17	•18	13	• 0 8	.11	• 00	• 0 0	• 0 0
12	.46	·!·	1.40	.00	60	•00	.36	•36	.14	• 00	c0	.25	• 0.0	940
13	• 0 0	.00	08	•00	.39	c0.	•01	40	.27	.02	.25	.20	14	R0.
1	.14	•00	e0.	• 00	06	• 02	.17	.10	1	• 25	+2°	.92	•24	• 0.0
15	.31	.00	50.	.00	+1	• 00	.12	•13	• 26	• 02	11	.18	• 0.8	• 0 0
16	.13	• 00	CE	• 00	- 22	• 0 0	11.	•13	• 10	10-	22	•15	• 0.0	.02
17	11	• 0.0	53	.00	EU	• 0 0	• 22	• 05	.02	• 00	10.	• 05	16	• 00
14	10.	.00	.15	• 0 0	56.	-02	•00	• 02	.16	• 00	·13	• 00	.16	· 02
61	.28	.00	02	.00	۲, z	.02	*0 *	• 00	.41	•13	00.	• 00	•19	• 08
1.2	08	• 02	- 02	• 0 0	.01	-00	*0*	c0.	02	.10	10.	• 00	.35	· 02
12	+0.	.00	.58	• 0 Đ	.65	•56	12	• 00	.31	.07	.14	.00	•14	.00
22	.61	P0.	02	• 0 4	60	• 30	10.	• 00	. 45	•13	.11	• 00	16	• 0.0
53	41	.13	. 35	· 05	دي .	• 00	.12	• 02	.13	• 00	P0.	• 00	• 0.6	• 23
52	01	.00	.18	• 0.0	1.01	• 02	.21	• 02	• 00	• 00	•.15	• 0.0	.20	• 53
55	.21	00.	. 45	- 02	• 64	.86	• 23	• 02	• 25	•15	÷0*	• 00	• 0 0	-05
55	07	.02	00.	• 0.5	• 0.9	.51	• 38	.30	. 62	• 38	.21	• 00	• 08	• 0 0
27	1.00	·15	.02	• 00	80°-	• 02	•29	•10	• 40	64.	.4A	• 00	10	.18
54	•14	.81	• 18	• 0.0	.00	• 00	*.16	•08	.24	10.	1.10	•38	• 0 •	• 63
62	• 60	• 28	15	• 00	+ C • -	• 00	• 02	• 00	.25.	• 25	.15	.25	.01	.25
30	++ °	٤ć.	.25	• 00	.21	• 00	.13	• 00	• 00	• 00	.11	.02	60*-	.02
31	22	62.	.00	• 0.0	.30	• 22	* 34	• 00	• 00	• 00	°28	.20	• 0.0	• 00

IAULE 5. DAILY VALUES OF P-E AND P G FOM WATEN YEAR 1961. (UNITS--CENTIMETERS OF WATEN DISTRIBUTED OVEN THE BASIN

WATEH YEAR 1961

	001	DBER	NUVEN	ANER	DECEN	1JER	JANL	AHY	FEBR	JAHY	MP	RCH	A	HIL
UAY	P-E	ьЧ	p-E	9 a	u-d	D d	9-4	d d	9-d	ŗ	3-d	æ	ш-d	£
1	.3H	•00	38	-02	12.	• 0.0	02	= 0 A	10	• 00	11.	• 05	02	c 0.
N	.12	-02	•00	.00	.32	.10	16	• 00	.07	• 02	+c.	.02	÷0	• 05
ŝ	.30	-02	.96	.13	1.06	.10	• 05	• 0 0	EE.	•13	.40	•66	••08	0.0.
\$	•2*	-02	.80	•15	.60	•5.	.15	.00	.22	• 05	.40	.23	• 4 3	20.
r	57	• 00	11	- 02	.30	.23	-+05	• 0.0	24	• 00	00.	.33	56	• 0.0
s	•56	• 00	14.	.18	. 14	<0·	• 08	• 00	26	• 00	.80	• 69	05	00.
1	E0.	• 02	19.	.69	05	• 00	•00	• 00	10	• 00	•• 08	•14	.15	.61
τ	.11	D**	- 02	.50	02	• 02	15	• 00	60.	• 00	16	00 .	•36	•18
6	1.31	1.37	11	•09	.37	•15	02	• 0.0	.16	-02	.12	.00	10	¢1°
0.1	1.05	• 69 •	.01	.00	22	۰l.	05	•00	.37	.10	00.	-05	[+ *	C 4 .
11	•8.	· 45	11	• 00	RT	.00	10.	• 00	د۱.	• 00	.40	.10	15	• 0.5
12	.37	•13	.68	• 0.2	. 62	• 02	02	• 00	.37.	.10	02	.15	.12	• 0.0
13	.41	c 0.	11.	• 02	.00	.00	÷0	.00	.20	50.	30	.00	• 23	c2.
41	ċ0.	.18	.54	•13	11	.00	<0°-	•00	10	• 00	.20	• 00	+0 •	.13
51	c0	• 23	. 20	•61	- 15	• 0 •	15	• 0 0	.01	• 00	.30	00.	10.	.02
16	.20	• 33	. 15	• 02	.22	• 00	÷0.	• 0.0	· 45	• 00	+	.20		• 0.0
17	. 58	.27	.01	• 00	. 28	.13	• 02	• 0.0	.37	•18	10	·02	- 4 ()	• 0 0
14	12.	.48	۶ ۶ •	• 02	60.	دا ،	05	• 00	10.	.21	10.	.13	• 50	• 0 0
6.1	01	•00	14	80.	• 00	.00	19	• 00	P1	• 00	. 45	.20	.4.8	·1.
50	01	• • 0	.11	.00.	10	• 02	38	• 00	26	• 02	.25	.10	16.	c1.
51	01	• 00	e£.	• 00	• 03	50 .	08	00.	P0.	• 00	.20	• 05	\$.	• 0.0
22	20	• 00	02	• 00	.26	• 0.0	+0+-	• 00	.33	.20	05	.00	1 +	s0.
53	01	• 0.0	.16	.00	• 00	• 00	•00	• 00	.10	.10	.17	• 0.0	• 00	-05
54	• 05	•00	12	• 0.0	.21	• 0 0	• 02	• 00	• 01	• 0.0	.40	.18	•31	.10
25	.50	• 00	140	• 0.0	20	• 00	• 03	.15	.21	•15	12.	• 33	10.	.10
44	÷03	• 00	.56	• 00	.39	40 .	• 31	• 20	.32	• 25	65.	.30	•13	• 0 0
21	12	• 0.0	• 76	.38	10	•13	• 36	• 33	•00	• 00	82°	.23	•08	• 00
82	.20	÷09	20	• 20	02	.10	08	• 00	.17	• 15	.11	64.	16	•00
62	د۱.	۰1۵	80°	• 00	03	• 00	17	• 00	• 00	• 00	.13	.33	• 00	• 0.0
30	11	• 00	•00 •-	• 02	¢0.	• 00	.10	• 0.0	00.	• 00	20.	• 08	16	• 0.0
16	• 0.2	• 00	.00	• 00	.25	• 25	• 15	• 0.0	.00	• 0.0	- 20	• 02	• 0.0	• 0.0

IABLE 5. DAILY VALUES UF P-E AND P FOR WATER YEAR 1962. (UNITS--CENTIMETERS OF WATER DISIRIBUTED OVER THE HASING

	100	OBER	NUVE	MBER	DECEN	MBER	NAU	UAHY	FEBRI	JAHY	Ψ	HCH	A	HIL
UAY	P-E	PG	7-4	9 a	J-d	D 4	H-G	D 4	P-F	2	P-F	2	J-d	a
T	.43	· 05	.34	•10	0 1 .	• 05	02	.00	- 08	.00	×0.	• 0.0	0+*-	• 0.0
ν	0 * * -	• 00	10	• 33	.80	.02	08	.00	•03	•00	.09	• 00	25	• 0.0
T	.08	• 00	-11	• 00	.49	•15	e0.	• 0.0	c0.	.00	•34	• 30	.20	• 0.0
4	21.	00.	10.	•0.4	11	.10	35	- 02	•10	• 0.0	•0•	.02	15	c0.
r	.18	• 00	1/	• 0.0	e5.	• 0.0	10	00.	• 0.0	• 0.0	03	• 00	E0.+	80.
s	.14	• • • •	15	• 00	.03	• 05	+0	.10	.10	• 0 0	•19	• 00	27	•10
	.19	.14	04	• 0 0	LJ	• 00	25	•35	.60	•13	.58	.33	• 25	• 10
r	.10	96.	•10	• 0.0	.30	•13	30	64.	\$4.	• 64	86.	.10	.35	*20
5	1.21	56.1	01	• 02	15.	.18	+0	•15	c'U .	.61	-11	¢0.	•14	12.
10	. 74	.13	cl	• 0.0	.76	.27	11	• 00	11.	• 25	.15	•25	13	.15
E	c,0.	50.	°.08	-02	.14	¢0.	10.	• 00	.62	·1.	• 54	.18	20	• 9.0
2	.63	ć1) .	35	•15	.00	•00	.20	.00	1.20	• 33	00.	*u2	EU	• 0 ()
13	45	• 00	60	•00	.10	• 00	• 35	•30	. 45	.45	• 0.0	.02	-05	• 0 0
14	20.	• 0.0	02	•00	.21	• 04	+0	.10	• 26	.05	09	.00	20	• 0 0
12	.31	.00	HE.	•00	.80	.20	P.0.9	P0.	• 34	SU.	09	• 0.0	11.	• 0.0
Ľ	c1.	00-	66.	-05	.1н	12.	10	• 0 0	+1.	٤ć.	08	• 00	36	• 0.0
11	.22	• 0 0	03	12.	.18	.20	•20	• 0.0	10.	.33	05	• 0.0	•01	• 00
11	.11	• 00	SU.	¢0.•	11	.27.	.11	.20	.08	· 0.2	• 0.6	• 00	+2	• 0.0
51	.06	.00	£1.	20-	E0.	.13	• 65	•50	. 44	• 0H	10.	• 0.0	• 08	• 00
511	c2.	• 0 0	.15	.13	.31	• 20	· 35	1.20	• 35	.45	.29	•08	.15	• 0.0
ر ک	.47	• 0.0	.08	.51	.30	• 08	11.	1.02	.13	.40	- 02	. 15	.11	G 0 =
22	.02	• 30	.29	•10	•00	• 05	•16	.10	·15	.10	. J	•10	32	• 0 ()
53	9F.	•09	+0.	= 02	13	•00	• 00	.00	.40	• 25	.25	.35	01	• 0 0
54	.42	•00	• 50	• 00	.15	.00	•10	• 00	. 63	•18	. 3U	• 05	06	.00
52	.13	•00	.10	• 25	.15	• 02	• 02	• 00	• 33	12.	24	• 00	.78	• 00
47	.71	• • •	.10	• 33	.05	• 02	+0 ·-	• 00	¢9*	• 25	06	• 00	.15	•51
12	.62	• 35	07.	• 00	60	•13	60°	• 0 0	•09	P0.	+0	• 00	0 * * -	·05
HN	.68	949 .	05	.00	24	• 00	60*-	• 00	• 0.6	• 00	.40	• 0.0	• 35	6c.
67	.42	.18	GE .	• 0.0	.10	•15	• 00	•00	* 0.0	• 00	• 00	• 00	•20	• 33
UF.	.19	.59	ç2.	SO.	14	• 02	05	• 00	• 00	• 00	01	• 00	17	• 02
15	36	.33	0.0 -	.00	ET	00-	61.	00-	- 00	-00	08 -	100	000	.00

IABLE 5. DAILY VALUES UP P-E AND P_G FOR WATER YEAR 1963. (UNITS--CENTIMETERS OF WATER DISTRIBUTED OVER THE BASIN)

WATEH YEAR 1963

	001	UHER	NOVEN	ABER	DECEN	ABER	JANL	YHAU	FEBHL	1AHY	W	RCH	A	JING
UAY N	p-E	с 1	7-4	e G	B-e	ť	a-d	ť	3-d	ť	9-E	Ŧ	9-E	a.
1	57	.00	04	.00	.10	.15	13	•00	10	66.	90	.10	.15	00.
N	22	• 00	26	.00	10.	00.	07	• 00	.45	•18	.28	80.	. 38	• 33
m	E0.	50.	05	• 00 •	10.	50.	• 00	•15	.25	.13	.35	.33	02	10.
4	1.10	.20	.20	• 0.0	•.25	.13	•16	s2.	.15	• 0.2	.01	.18	.10	00.
'n	.58	1.01	05	• 0 0	15	• 00	17	• 0.0	50 .	• 00	· 08	• 02	35	11.
ç	e0.	.18	·.09	• 00	.15	.00	• 0.8	• 00	05	• 0 0	18	• 0.0	60.	· 02
1	.24	.00	63	• 0 u	27	• 00	27	• 00	10	• 0.0	r0*	.00	16.	.16
r	.32	• 0 0	.18	• 0.0	-,25	• 00	.02	• 0 0	35	• 00	06	• 00	-15	99.
ъ	20	.00	• 00	• 00	03	• 0 0	.22	• 00	+6	• 0.2	.18	• 00	• 6 ()	c1.
10	.12	.00	15.	c () •	.17	• 00	.32	•9.	• 00	.20	• 06	• 00	.60	00.
11	+2	0.0.	07	· 02	32	• 0.0	• • •	• 08	.35	05.	0 * *	.10	+0*-	-02
12	20	.00	.13	.00	• 0.0	.00	- 02	.00	05	• 02	.20	· 35	10	• 0.0
13	18	- 02	10	• 0 0	•.00	.00	.10	• 0 0	• 02	.00	15	c0.	+0	• 0 0
14	2 ⁵	.08	1.20	•22	.20	• 00	•0 •	- 02	00.	• 00	.15	.00	60·	0 0 ·
51	37	.20	* 2 A	.53	10	.00	-14	• 05	.01	• 05	.67	.10	.37	• 1 •
16	.50	643	.41	•13	.18	.00	.23	• 08	- 02	- 02	.10	.20	19.	.13
21	.70	FC.	11	.23	61.	• 0 0	.18	.10	.32	• 00	.55	.15	•56	•10
81	. 71	• 25	.34	. 23	.28	.13	•50	• 59	.12	• 0.9	.04	. 91	1 + •	.30
19	.33	• 10 10	.22	.00	20.	.21	•16	•15	.17	.18	06	. 02	• 36	c0.
50	1.45	.10	44.1	• 00	-,15	~ 0 S	• 01	• 00	.31	.20	.03	.00	15.	÷ [•
17	•00	.00	30	.00	10	• 00	• 4 0	• 0.0	•19	• 05	.00	• 00	16.	40.
22	1/	• 0 0	+0.	• 0.0	27	• 02	10	• 00	23	• 0.0	c£.	• 00	.12	50.
53	28	• 00	·00	• 00	.17	.00	20	• 00	14	.00	.05	• 00	• 0 •	.00.
42	80.1	• 00	06	• 00	.32	.43	• 0.5	• 00	.00	• 00	.25	• 05	.15	0.0 .
52	.11	• 00	.37	-02	•0•	÷0.	.25	•18	18	- 0.2	- 15	.00	14	.00
56	.31	• 00	٤٤.	-02	.05	• 00	16	• 02	.40	• 05	• 0 •	• 00	1.00	.23
12	41	• 00	*0*	• 00	. 14	• 0.0	• 02	.00	.14	• 30	35	.00	.70	55.
54	31	• 00	.14	- 05	10*-	• 00	+0*-	• 00	10.	.02	.06	.40	.15	.42
53	.18	• 00	.20	• 0 1	10.	• 00	*0*	• 23	• 00	• 00	06	• 4 0	35	• 0.0
30	.07	• 00	.28	• 00	• 36	• 00	•50	• 29	.00	• 00	.10	• 00	37	.00
31	10	.00	.00	• 00	01	.00	1.18	66.	.00	• 00	32	• 0 0	• 0.0	.00

53



Figure 16a. Daily time series and three-day running averages of P-E (solid lines) and P_G (dashed lines) for the winter season of water year, 1957.



Figure 16b. Daily time series and three-day running averages of P-E (solid lines) and $\rm P_G$ (dashed lines) for the winter season of water year, 1958.



Figure 16c. Daily time series and three-day running averages of P-E (solid lines) and P_G (dashed lines) for the winter season of water year, 1959.



Figure 16d. Daily time series and three-day running averages of P-E (solid lines) and P_G (dashed lines) for the winter season of water year, 1960.



Figure 16e. Daily time series and three-day running averages of P-E (solid lines) and P_G (dashed lines) for the winter season of water year, 1961.



Figure 16f. Daily time series and three-day running averages of P-E (solid lines) and P_G (dashed lines) for winter season of water year, 1962.



Figure 16g. Daily time series and three-day running averages of P-E (solid lines) and $\rm P_G$ (dashed lines) for the winter season of water year, 1963.

smaller precipitation, however, often show large discrepancies between the P-E and P_G values, with P-E values consistently larger than the PG values. The good correspondence between the two parameters for the very large precipitation events is a reflection of the ability of the radiosonde network to sample the intense synoptic scale systems producing these large precipitation events. In these instances the orographic influence on precipitation is suppressed due to the dynamically produced vertical motions over the basin. The apparent persistent discrepancy between the precipitation estimate and water balance computation on days with small basin precipitation is interesting. Two possible explanations can be put forth. First, the excess of P-E over PG could be due to systematic errors in the evaluation of the atmospheric water balance equation due to the neglect of the vertical eddy flux term or to the existence of the stationary cloud system on one boundary of the basin. As pointed out in the last chapter, the evaluation of these possible errors depends upon data not presently available. Secondly, the deviation could reside in a systematic underestimation of the actual precipitation by the PG values. For the conditions during the winter over the large mountainous area under consideration, this source of systematic error can be extreme.

Some data demonstrating the increase of precipitation with elevation for a local area in the central Rocky Mountains were available to the author through the courtesy of Professor L. O. Grant. These data consist of the measurement of the water content of snow fall using snow boards as the sampling device. Sixty-three snow boards located at various elevations over three passes in central Colorado are included in the sample. The data for several precipitation periods totalling 103 days were assembled and grouped according to elevation class; and then the average precipitation from the snow board data for each class was compared to the $P_{\rm G}$ data for the same periods. Table 6 gives the snow board measurement expressed as a percentage of the P_G value along with the percent of area of the basin having elevations within the class interval. Let us assume that this profile of precipitation amount with elevation can be applied to the entire basin. Then one obtains the following relationship for the P_G data corrected for bias due to the distribution of gauges with elevation.

$P_{GH} = 1.20 P_{G}$

Here P_{GH} denotes the corrected estimate of basin precipitation P_{G} .

TABLE 6

The Snow Board Measurements Expressed as a Percentage of the Precipitation Gauge Data P_G for Various Elevation Classes. Also Shown is the Percentage of Area of the Basin for Each Elevation Class

Elevation Class (Ft. Msl)	Snow Board Measurements (% of P _G)	Percent of Area of Basin
8000 - 9000	115	10
9000 - 10,000	115	8
10,000 - 11,000	175	6
>11,000	250	3

This analysis, while not conclusive because of the generalization assumed for the total basin from very local data, demonstrates the magnitude of the bias due to the gauge network.

Because of these problems, a statistical evaluation of the daily series is tenuous and thus not presented here. The conclusion to be reached from the daily data is that the daily trends observed by the precipitation gauge network are reflected by the water balance
computation and that the correspondence is particularly good during large precipitation events and also during periods of extended dryness.

Seasonal Atmospheric Water Balance

The wintertime water balance of the Upper Colorado River Basin was obtained by accumulating over each winter season the data presented in Table 5. The seasonal values for P-E and P_G for each of the seven winters are listed in Table 7.

It is of interest to evaluate the relationship between the seasonal basin precipitation estimate P_G and a precipitation measure determined solely from the atmospheric water balance results. To this end it is convenient to define a minimum seasonal basin evaporation, E_{min} , as the accumulated sum of the parameter P-E on those days each winter when the result is negative. Stated another way, this minimum seasonal evaporation is the evaporation computed assuming there was negligible evaporation on all days when the precipitation exceeded evaporation and also that there was negligible precipitation on all days when the evaporation. It follows that a minimum seasonal precipitation, P_{min} , then may be defined as:

$$P_{min} = (P - E) + E_{min}$$

Table 7 also lists the seasonal values of ${\rm P}_{\min}$ and ${\rm E}_{\min}$ for each of the seven winters.

Figure 17 shows both the seasonal P-E (triangles) and P_{min} (dots) plotted against the seasonal precipitation estimate P_{G} . The correlation between the parameters yields coefficients r = 0.7 and r = 0.9, respectively. Because of the small sample size, further statistical evaluation was not warranted.

TABLE 7

Water Year	P _G	P-E	P _{min}	\mathbf{E}_{min}	
1957	32.2	39.2	47.9	8.7	1
1958	28.0	20.8	34.2	13.4	
1959	18.9	23.0	32.9	9.9	
1960	22.6	28.2	37.3	9.1	
1961	20.9	24.5	35.8	11. 3	
1962	25.1	30.1	40.3	10.0	
1963	20.3	18.0	32.3	14.3	
Average	24.0	26.3	37.3	12.4	-

Seasonal Values (cm)



Figure 17. Seasonal precipitation estimate P_G plotted against seasonal P-E (triangles) and P_{min} (dots). The dashed line is that line passing through the origin having a slope equal to the ratio of the average seasonal values of P_{min} to P_G .

The dashed line entered in the diagram is the line constructed to pass through the origin having the slope of the ratio of the average seasonal value of P_{min} to the average seasonal value of P_{G} . This line describes the approximate relationships between the calculated and observed seasonal precipitations.

$P_{min} = 1.5 P_{G}$

The verification of this relationship is most difficult to obtain because of the uncertainty due to the small sample size and the large number of sources of error in the P_G data. Based on the results of the snow board data presented in the preceding section, a factor of 1.5 seems high. If one considers possible errors due to the wind affecting gauge catchment in addition to the neglect of the high elevations in the P_G data the discrepency between the factors 1.5 and 1.2 diminishes. Finklin (1967), further, has shown that there are a number of shielded storage gauges located in various parts of the alpine region of the Colorado and Wyoming Rockies that have an average winter season catchment of over 120 cm of precipitation. Precipitation of this magnitude distributed over a large enough area would explain the factor of 1.5. Unfortunately, not enough data is available to pursue this point further.

It is interesting to note that the accumulated seasonal P-E data plotted against seasonal P_G data as shown in Figure 17 are scattered about the line of perfect relationship, thus the apparent underestimate of precipitation as measured by the gauge network is approximately equal to the minimum basin evaporation. The relationship of the seasonal atmospheric water balance to the annual runoff from the basin will follow in the succeeding chapter.

The Monthly Atmospheric Water Balance

The monthly atmospheric water balance was obtained by the summation of the daily P-E values for each month. Because the

river basin stores much of the accumulated water in the snowpack over the winter season, the runoff cannot be used as an independent check of this quantity. It is of interest, however, to evaluate the relationship between the monthly precipitation estimate P_{G} and the results of the atmospheric water balance computation. Table 8 lists the monthly values of PG, P-E, Pmin and Emin along with N_E , the number of "evaporation" days for each month. Also given are average values for each parameter for each of the seven months. There appears to be no strong seasonal trend in any of the parameters except, as one would expect, in the E_{min} values which show a minimum from January to March with a maximum in October and April. Figure 18 shows the monthly basin precipitation P_G plotted against monthly P-E. The dashed line entered on the diagram indicates perfect agreement. A wide scatter about this line is noted. If, however, one plots the monthly PG values against monthly Pmin values, (Figure 19), only four months show ${\rm P}_{\rm G}$ to exceed ${\rm P}_{\rm min}.$ The correlation coefficient for this relationship is r = .6 and the line of perfect agreement is again entered as the dashed line. persistent excess of P_{min} over P_G re-emphasizes the conclusion reached for the seasonal case, that P_G is a gross understatement of the actual precipitation occurring over the Upper Colorado River Basin.

The Natural Period Analysis

Given an accurate measure of the basin precipitation, the daily course of evaporation from the basin could be obtained directly. Following the analysis presented in the preceding sections, it is evident that the basin precipitation estimate derived from the rain gauge is not suitable. The study by Riehl and Els berry (1964) shows that the precipitation regime of the Upper Colorado Basin is dominated by the occurrence of large precipitation episodes.

WATER	AN MONTO	12 au	10.00			ALC: NO
WAICH TI	LAN MUNIH	P G	PEC	MIN	MIN	E
1457	10	3.4	11.4	13.0	1.2	5
	12	3.7	1.4.7	5.0	1.0	B
	1	9.3	0.9	7.0		8
	2	2.9	6.4	0.9	. 5	5
	3	4.5	4.2	5.0	1.5	9
		0.4	2.2	4 . D	2.4	10
1958	10	5.5	3.8	6.7	2.9	13
	11	4.9	3.8	5.3	1.5	10
	12	3.7	4.1	5.5	1.4	9
	4	1.9	• 9	3.0	2.1	13
	4	4.2	2.0	4.4	1.8	12
	3	2.9	1.0	4.2	2.4	12
1959	10	1.3	4.3	5.9	1.0	10
	11	3.4	4.9	5.4	.5	1
	12	1.9	2.3	3.0	1.3	11
	ż	4.8	6.0	6.0		
		2.2	2.3	3.1	1.4	12
		3.0	3.0	4.3	1.0	10
1960	10	4.4	5.7	7.4	1.1	10
	11	1.1	2.9	4.4	2.0	11.
	12	2.H	3.2	4.1	1.0	15
	4	2.6	3.5	4.4	• 9	5
	c i	4.7	1.0	1.9	• 0	2
		2.9	1.0	2.9	1.4	9
1124444	10.0 × 10.0 ×			Sales and the		
1961	10	5.0	5.5	7.3	1.8	10
	11	3.3	0.9	1.1		
	12			1.0	1.7	17
	2	1.8	2.8	4.0	1.2	7
	3	5.0	4.3	6.0	1.1	.9
	٠	5.4	2.1	4.5	2.4	12
1962	10	4.6	8.1	9.6		2
	11	2.5	2.4	4.2	1.9	12
	12	R.5	5.2	6	1.1	A
	1	4.1	2.2	4.0	1.1	15
	ć	6.1	9.1	9+1	• •	1
	3	2.0	3.6	2.7	1.1	15
	1751	6.4.90	⊙• ⊀		4.3.4%	10
1903	10	1.5	1.5	5.5	۹.D	15
	11	1.5	2.0	4.1	2.5	15
	12	1.5	. 5	2.4	1.3	14
	3	2.4		2.2	2.4	13
	3	3.3	3.4	4.5	1.1	н
	4	3.7	6.3	7.9	1.0	4
	ant at a		1	40.005		
	AVERAG	E 9019	HUT V	ALUES		12217
	10	4.6	5.9	1.7	2.0	4
	11	2.7	3.0	5.0	1.2	10
	1	3.5	2.8	4.2	1	11
	e	3.9	4.d	5.0	1.9	1
	3	3.7	3.1	5.0	1.3	1.0
	60	3.4	2.3	*.2	2.2	11

TABLE B. MONTHLY VALUES (CM). PGIS BASIN PRECIPITATION FROM GAUGE DATA. PPE IS RESIDUAL FROM MATER BALANCE. PHINAND EMINARE MINIMUM VALUES DERIVED FROM PPE DATA. NE IS THE NUMBER OF EVAPORATION DAYS.



Figure 18. Monthly P_{G} plotted against monthly P-E. Dashed line is the line of perfect agreement.



Figure 19. Monthly ${\rm P}_G$ plotted against monthly ${\rm P}_{min}$. Dashed line is the line of perfect agreement.

These episodes are separated by periods of little or no precipitation. The atmospheric water balance computation allows one to extend this type of study and investigate the periods of little or no precipitation in order to determine the evaporation occurring over the basin during these periods. To this end the daily series was divided into "natural periods" showing homogeneity in the parameter P-E.

The attractive feature of such an analysis is that the important evaporation and precipitation events, lasting more than one day are dealt with, something largely dissected if one deals with daily values and something largely glossed over if one deals with arbitrary chronological divisions such as weeks or months.

Definitions of natural periods: The daily series of P-E data were conveniently broken into three distinct groups, storm periods, net precipitation periods and net evaporation periods. The limits determining each class are as follows:

A. Storm Periods:

B. Precipitation

Periods over which the accumulation of positive P-E data was 1.00 cm or greater under the requirement that the average daily value over the period exceeded .25 cm. The storm period was terminated if the daily value was less than .10 cm. Single days were counted as storms if the P-E value on that day exceeded .50 cm.

Periods: Periods other than the storm periods over which the accumulated P-E was positive under the requirement that no two consecutive days had negative P-E values.

C. Evaporation Periods: Periods over which the accumulated P-E was negative. In this summation no more than two consecutive days are allowed to have positive values. The period must begin and end with negative values.

Table 9 is an example of the classification of the daily data into natural periods. Also included in the table are the corresponding

daily values of the P_G data. These data were grouped according to natural periods by simply summing over the same time interval as dictated by the P-E series but allowing for a variation of no more than one day at either or both ends of the period. This variation was imposed in order to account for the inconsistent times of observation of the free atmospheric data and the gauge precipitation data.

The seasonal analysis of the natural periods: Table 10 gives the complete chronological set of natural periods covering the seven winters including the accumulated P-E and P_G for each period, the starting date of each period, and its length. The periods beginning and ending the time series for each year cannot be explicitly defined and, therefore, carry a code 4 under the heading "type of period."

Table 11 summarizes the natural period analysis for each of the seven winters. Included in the table are the accumulated values of P-E and P_G for each of the three classes of natural periods along with the number of periods included in each class.

Figure 20 protrays the data of Table 11. Here the seasonal accumulation of water over the basin is plotted against the seasonal accumulations of the three types of periods. It is apparent from these diagrams that the variability of seasonal accumulation over the basin is largely described by both the storm and evaporation periods. Little of the variability is explained by the net precipitation periods. This result is compatible with that of Riehl and Elsberry (1964) and Marlatt and Riehl (1963) for the precipitation regime of the basin; however, it also shows the effect of periods of dryness. The years with low water accumulation are characterized by greater evaporation occurring during periods of negative P-E. Again because of the small sample of seasonal values, no statistical evaluation of this data is merited. A more definitive treatment of the storm and evaporation periods will follow in succeeding sections.

Ľ Yr.)ate Mo.	Day	P-E (cm)	Type of Period	Total P-E Over Period (cm)	Av. Daily Rate (cm/day)	P _G (cm)	Total P Over Period
61	3	23 24 25 26 27 28 29 30	.17 .42 .27 .53 .28 .11 .13 .08	STORM	2.02	. 25	0 . 18 . 33 . 30 . 23 . 43 . 33 . 02	1.88
61	4	31 1 2 3	20 02 09 08	EVAPORATION	39	10	.02 .05 .02 0	. 09
61	4	4	.46	PRECIPITATION	. 46	. 46	. 02	. 02
61	4	5 6	56 05	EVAPORATION	61	30	0 0	0

TABLE 9 Example of the Natural Period Determination Scheme

TABLE 10.NATURAL PERIODS FOR WATER YEAR 1957. THE CODE FOR -1YPE OF PERIOD-IS 1=SIORM,2=PRECIPITATION,3=EVAPORATION,4=UNDEFINED. UNITS FOR P-E AND P. DATA ARE CM OF WATER DISTRIBUTED OVER THE BASIN G

PERIUD	DATE	0F	START	TYPE OF	P=E	Ρ.	LENG	TH OF
NUMBER	OF	PE	RIUD	PERIOD	(CM)	(CM)	PER	RIOD
	MON	Id-	DAY		North Carlot	1997 (2003) A. F. F.	P=E	P
1	1	0 -	1	4	1.27	.13	2	1
2	1	- 0	3	14	55	.00	1	2
3	i i	0 -	4	ĩ	1.06	.00	2	2
4	ĩ	0 -	5		15	.00	ĩ	2
5	ĩ		7	1	3.93	.38	Ä	7
6	ĩ	0 -	15	2	.46	.00	2	2
7	- î	0 -	17	3	46	.00	3	2
8	- î	0 -	20	1	. 65	.02	ĩ	2
9	- î.	0 -	21	2	-16	- 0.0	ŝ	2
10	î.	0 -	23	5	6.09	3.10	1.0	15
11	i		2	2	1.66	1.75	25	24
12	î	1	27	2		.00	- 3	24
13	1		30	2	.20	.00	ĩ	3
14	1		1	2	- 44	.00		1
15	1	2 -	4	3	4.85	3.45	10	10
16	1	5	14	2	1 03	31	1 14	10
17		5 -	1	2	1.73	1 13	10	11
19		10	2	1	1.73	1+12	2	b
10		1	0	3	38	.05	2	1
19		1 7	8	1	.83	1.15	÷.	1
20			4	1	1.38	2.00	2	6
21		1 -	14	3	22	.20	*	4
22			18	2	.95	.00	1	5
23		1 -	C D	1	2.08	3.32	0	7
24		1 -	31	3	10	.00	1	-0
25		- 5	- to	1	2.58	1.16	8	10
26	16	5 -	9	2	.17	.02	2	1
27	2	2 -	11	3	15	.00	2	5
58		2 -	13	2	.19	.00	2	5
59	Ĵ.	5	15	3	03	.00	1	1
30	1	2 -	10	1	3.45	1.10	10	11
31	- 1	2 -	20	S	.37	.31	4	3
32	- 8	3 -	4	1	. 50	. 51	1	5
33		- 6	3	2	• 4 1	• 96	3	4
34		3	0	3	56	. < 3	3	3
35		3 -	9	1	2.02	1.55	5	4
30	- 3	3 -	14	5	. 52	.25	1	7
31	6	3 -	21	1	.58	.53	1	5
38		3 .	22	2	.10	.00	1	1
39	1	3 -	23	3	44	*05	4	5
40		3 .*	21	2	.08	.15	2	4
41		3 -	29	1	1.44	1.31	5	5
42		4 -	3	3	25	.05	2	1
43	10	4 -	5	1	1.17	1.51	3	3
44	3	4 -	В	5	.24	.00	2	1
45	.0	4 -	10	З	-1.33	.45	6	6
46	2	4 -	16	S	.26	. >1	4	5
47	10	4 -	50	3	57	.00	2	2
48		4 -	55	S	.81	1.88	6	5
49		4 -	28	4	1	22	4	1.

TABLE 10.NATURAL PERIODS FOR WATER YEAR 1958. THE CODE FOR -TYPE OF PERIOD-IS 1=STORM,2=PRECIPITATION,3=EVAPORATION,4=UNDEFINED. UNITS FOR P=E AND P DATA ARE CM OF WATER DISTRIBUTED OVER THE BASIN

PERIOD	DATE	OF	START	TYPE OF	P-E	PG	LENG	TH OF
NUMBER	OF	PE	2100	PERIOD	(CM)	(CM)	PE	COLE
	MON	THe	YAC				P-E	P
1	1	0 -	1	4	.20	. 53	3	4
2	1	0 -	44	3	-1.23	.18	5	5
3	1	0 -	9	1	1.00	1.32	4	4
4	1		13	1	.77	.94	1	1
5	1	0 -	14	3	70	.10	2	î
6	1	0 -	16	2	. 39	.02	2	1
7	1	0 -	18	1	2.58	2.14	6	8
8	1	0 -	64	3	35	. 00	2	2
9	1	0 -	20	1	1.10	.23	3	3
10	1		29	3	45	.00	2	1
11	1	- 0	31	2	.45	.00	2	1
12	1	1 -	2	1	2.95	2.69	5	7
13	1	1 -	7	2	.23	.10	5	4
14	1	1 -	12	1	1.03	1.09	3	4
15	1	1 -	15	2	.15	.38	2	S
16	1	1 -	17	3	61	.50	7	7
17	1	1 -	24	2	. 39	.05	3	3
18	1	1 -	21	3	51	.09	6	6
19	1	5 -	3	1	1.37	1.19	. 4	4
20	1	s	1	3	81	.05	7	7
21	1	2 -	14	1	2.30	1.40	6	6
22	1	- 2	20	2	1.37	1.07	12	11
23		1 -	1	3	65	.00	9	9
24		1 -	10	2	.42	.16	4	5
25		1 -	14	3	47	.00	3	2
26		1 -	17	5	.09	.00	÷.	1
27		1 -	10	1	. 54	.23	1	5
28		1 -	14	5	.12	.05	1	1
29		1 -	20	3	25	.00	3	3
30		1 -	23	5	. 35	. 99	1	5
31		1 -	30	1	.05	. 46	1	3
32		1 *	31	5	.08	.02	5	1
33		2 -	±	3		.00	6	S
34	1	2 -	3	1	1.07	.08		3
35		ć -	-A	3	</td <td>.10</td> <td>3</td> <td>2</td>	.10	3	2
30	1	2 -	10	2	*11	.00	2	é
31	3	< -	14	1	1.07	1.00		2
30	3	< -	17	2	- 90	. 53	-	
39	2	5 -	23	3		.00		2
40	3	5 0	23	Ę	- 11	.00	1	2
41	9	2 -	24	1	- 06	. 00	î	-0
42		50	25	2	1.01	. 42	2	3
44	5	2 -	21	3	- 15	.12	4	4
45	5	3 -	~ 3	2	1.40	1.13	12	10
46		3 .	15	1	1.05	1.47	3	5
47		3 -	10	3	39	.00	3	2
48		3 -	15	2	.00	1.54	8	7
49		3 -	27	ī	2.53	2.31	ð	9
50		4 -	0	3	-1.24	.62	16	16
51		4 -	22	ī	1.44	. 55	3	4
52		4 -	27	4	- 34	-14	6	5

TABLE 10.NATURAL PERIODS FOR WATER YEAR 1959. THE CODE FOR -TYPE OF PERIOD-IS I=STORM,2=PRECIPITATION,3=EVAPORATION,4=UNDEFINED. UNITS FOR P-E AND PG DATA ARE CM OF WATER DISTRIBUTED OVER THE BASIN

PERIUD	UATE OF START	IYPE UF	P=E	PG	LENG	TH OF
NUMBER	OF PERIOD	PERTOD	(CM)	(CM)	PE:	100
······································	MONTH-DAY	1. Sec. 7 & Sec. 17			P-E	P
1	10 - 1	4	. 54	.02	1	2
2	10 - 2	4	19	.00	1	1
3	10 - 3	ĩ	1.38	.02	5	3
	10 - 4	- î	2.00	.00	6	7
5	10 - 14	2	- 18	.00	4	4
2	10 - 14	2	15	- 0.0	i i	1
7	10 - 10	E I	.82	.20	1 i	2
	10 - 20	2	- 46	.00	1	2
0	10 - 23	1	25	.00	2	2
10	10 - 25	c	•25	. 60	1	2
10	10 - 25	1	- 41	.07		6
11	10 - 28	3	F . 41	- 33	3	0
12	11 - 3	2	• 24	.20	5	3
13	11 - 5	3	24		1	10
14	11 - 1	1	3.24	2.01	10	12
15	11 - 17	2	• 14	.00	0	0
16	$11 - c_3$	1	1.10	.3/	2	4
17	11 - 28	3	-1.43	.02	4	8
18	12 - 7	5	•13	.40	۲	4
19	12 - 9	3	85	1.36	8	6
50	12 - 17	2	. 32	.00	5	7
21	12 - 22	1	1.45	• 41	5	5
55	12 - 21	2	.10	.17	4	2
23	12 - 31	1	1.16	.25	4	5
24	1 - 4	3	12	.00	2	2
25	1 - 6	2	.82	. 51	5	5
26	1 - 11	3	49	.02	5	4
27	1 - 16	2	2.31	2.32	53	24
58	2 - B	1	.80	. 91	1	2
29	2 - 9	1	1.74	. 52	4	4
30	2 - 13	3	09	.00	1	1
31	2 - 14	1	1.89	1.00	5	5
32	5 - 19	2	. 19	1.30	ы	R
33	2 - 27	з	85	.06	8	6
34	3 - 7	2	.05	.53	6	7
35	3 - 13	1	.05	. 30	1	2
36	3 - 14	2	.63	.49	11	11
37	3 - 25	3	17	.02	2	1
38	3 - 21	2	.64	.18	٤	S
39	3 - 30	1	.50	. 64	1	3
40	3 - 31	2	.12	.00	5	4
41	4 - 5	1	1.03	1.09	4	5
42	4 - 9	3	44	.05	4	4
43	4 - 13	2	1.17	. 16	1	7
44	4 - 20	3	59	.05	0	5
45	4 - 20	4	1.04	1.04	5	5

TABLE 10.NATURAL PEHIODS FOR WATER YEAR 1960. THE CODE FOR -1YPF OF PERIOD-IS 1=STORM.2=PRECIPITATION.3=EVAPORATION.4=UNDEFINED. UNITS FOR P=E AND P DATA ARE CM OF WATER DISTHIBUTED OVER THE BASIN G

PERIUD	DATE OF	START	TYPE OF	P=E	PG	LENG	TH OF
NUMBER	OF PEP	0019	PERIOD	(CM)	(CM)	PEN	COIS
	MONTH-0	DAY				P-E	P
1	10 -	1	4	. 62	.89	1	1
2	10 -	2	2	.10	.48	1	2
3	10 -	3	3	41	.00	2	2
4	10 -	5	2	.70	.43	3	2
5	10 -	8	3	34	.00	1	1
6	10 -	9	1	.81	.21	1	2
7	10 -	10	3	03	.00	1	-0
в	10 -	11	2	1.80	.19	11	11
9	10 -	22	ā	.61	.21	1	2
10	10 -	23	3	•.28	.00	4	2
11	10 -	27	ĩ	2.30	2.02	4	6
12	10 -	31	3	36	.02	2	1
13	11 -	2	1	6.24	.86	3	3
14	11 -	5	2	.17	.00	6	6
15	11 -	11	3	-1.22	.00	9	9
16	11 -	20	2	.02	.00	1	1
17	11 -	21	ĩ	.58	.07	1	ż
18	11 -	22	2	1.67	.12	12	11
19	12 -	4	3	28	.00	2	2
20	12 -	6	2	.76	.19	8	9
11	12 -	14	3	38	.02	6	4
22	12 -	20	ž	.01	.02	1	2
23	12 -	21	ĩ	.65	.86	Ĩ	2
24	12 -	22	3	09	.00	ī	1
25	12 -	23	ĩ	1.99	1.41	4	4
26	12 -	27	3	42	.00	3	3
27	12 -	30	2	. 79	.58	5	6
28	1 -	4	3	*.11	.00	2	2
29	1 -	6	2	.43	.28	2	à
30	1 -	8	3	50	.00	1	1
31	1 -	9	ĩ	1.16	1.00	4	4
32	1 -	13	2	.84	.50	11	10
33	1 -	24	ĩ	1.11	. 52	4	5
34	1 -	28	3	10	.00	1	Ň
35	1 -	29	ĩ	1.54	.87	5	5
36	2 -	3	3	20	.00	2	ñ
37	2 -	5	2	.12	.05	ī	î
38	2 -	6	ī	2.25	1.72	5	6
39	2 -	11	ž	1.99	.79	14	13
40	2 -	25	ĩ	2.98	2.51	10	11
41	3 -	6	2	.00	.00	1	-0
42	3 -	7	1	. 65	.31	1	2
43	3 -	в	2	. 31	.02	2	ĩ
44	3 -	10	3	42	.25	3	â
45	3 -	13	2	. 49	1.30	2	3
46	3 -	15	3	39	.20	4	4
47	3 -	19	2	.19	.00	6	5
48	3 -	25	ĩ	2.65	1.08	8	9
49	4 -	2	3	14	.00	3	2
50	4 -	5	2	.73	.00	2	2
51	4 -	7	3	47	.00	2	1
52	4 -	9	4	1.06	2.65	22	23

TABLE 10.NATURAL PERIODS FOR MATER YEAR 1961. THE CUDE FOR -TYPE OF PERIOD-IS I=STURM+2=PRECIPITATION+3=EVAPORATION+4=UNDEFINED. JNITS FOR P-E AND PG DATA ARE CM OF WATER DISTRIBUTED OVER THE BASIN

PERIUD	DATE	OF	START	IYPE OF	P-E	PC	LENG	H OF
NUMBER	UF	PE	UOIH	PERIOD	(CM)	(CM)	PER	100
	MON	H+	YAC	0.5 0.543			P-L	P
1	10) -	1	4	.41	.06	5	5
2	10) -	6	1	. 50	.00	1	1
3	10		7	2	.03	.02	1	1
4	10		8	ī	4.20	3.58	7	8
5	10) -	15	3	05	.00	1	-0
6	10) -	16	î	1.05	1.08	3	3
7	10) -	19	3	-1.02	.00	9	9
в	10		28	2	. 35	.23	2	2
9	10) -	30	3	47	.02	3	4
10	1		2	ĩ	1.90	.28	3	2
11	1		5	3	11	.02	ĩ	ì
12	1	-	6	ĩ	1.30	1.51	٤	4
13	î		9	3	21	. 90	3	2
14	1	1	12	ĩ	1.13	.80	5	5
15	19		17	2	-54	.10	в	0
16	13		25	1	1.79	.58	3	2
17			24	- 2		.02	ä	2
1.4	1	5	1	3	2 62	1.07		5
10	1		7	1	- 07	1.07	2	5
19	1 4		0	3	07	.00	6	4
20	10		3.0	2	• 31	.30	÷.	3
21	14	•	10	3	04	.02	0	ь
22	14	•	10	2	. 39	. 28	3	2
23	14	-	14	3	22	.02	6	2
24	14	-	<1	2	.69	• 23	0	7
25	14	-	21	3	06	.10	3	3
20	11		30	2	. 30	. 33	2	5
21	2		1	3	98	.00	22	55
58		-	53	2	.78	.68	5	4
29	33	- 1	28	3	25	.00	2	5
30		- 1	30	2	.80	• 20	6	6
31			5	3	=.65	.00	د	3
32		- 2	8	2	2.11	.69	11	11
33			19	3	44	.02	2	3
34		•	21	5	1.05	. /0	7	6
35	- 3	? -	28	1	1.73	1.44	5	6
36		3 •	5	2	.00	.00	1	- 0
37		3 -	6	1	.80	.84	1	5
38		3 •	1	3	24	.00	2	2
39		3 -	9	5	.58	.27	3	3
40		3 -	12	3	32	.00	2	2
41		3 -	14	2	.50	.20	2	5
42		3 -	16	3	40	-02	2	1
43		3 -	16	5	.02	.48	5	6
44		3 -	23	1	1.99	1.88	в	7
45		3 -	31	3	39	.09	4	4
46		4 -	4	5	. 40	.02	1	1
47	34	+ -	5	3	61	.00	2	5
48		4 -	7	2	1.06	2.04	9	9
49	3	•	16	3	58	.00	2	2
50		4 -	18	1	2.11	.30	4	4
51	3		55	2	.1H	. 35	6	6
52	9		28	4	32	.00	٤	3

TABLE 10.NATURAL PERIODS FOR WATER YEAR 1962. THE CODE FOR -TYPE OF PERIOD-IS 1=STORM,2=PRECIPITATION,3=EVAPORATION,4=UNDEFINED. UNITS FOR P-E AND P DATA ARE CM OF WATER DISTRIBUTED OVER THE BASIN

PERIOD	UATE	UF	START	TYPE OF	P-E	PC	LENG	IH OF
NUMBER	OF	PE	100	PERIOD	(CM)	(CM)	PE	100
	MUN	1	DAY	68 8 8 M			P-E	P
1	1	- 0	1	4	.03	.05	2	3
2	10	- 0	3	1	2.96	2.21	в	7
3	11	- 0	11	2	.05	.00	1	-0
4	1	0 -	12	1	.63	.07	1	2
5	10	- 0	13	S	1.14	.00	10	9
6	.11	- 0	23	1	4.30	2.70	10	12
1	1	1 -	2	3	-1.50	.32	13	13
в	1	1 -	15	5	.08	.02	3	1
9	1	1 -	18	1	1.01	1.08	4	6
10	1.	1 -	22	2	. 33	.02	2	2
11	1	- 1	24	1	. 50	.25	1	1
12	1	1 -	25	2	.35	.33	4	3
13	1	- 1	29	1	2.35	.37	5	6
14	1.	2 +	4	2	.04	.05	4	3
15	11		H	1	1.59	.63	4	4
16	1	- 1	12	2	.00	.00	1	1
17	1.	2 -	13	1	1.59	1.02	5	6
18	11	2 -	18	2	.19	.63	9	9
19	13	2 -	21	3	-1.01	.19	10	9
20		•	6	3	74	1.09	5	6
51		- 1	11	5	.62	. 40	3	3
22		1 -	14	3	22	.08	3	3
53		- 1	17	1	2.84	3.08	b	5
24		- 1	23	S	.12	.00	3	3
25		1 -	20	3	11	.00	5	5
26		1 -	31	2	.29	.00	6	7
27	i i	5 -	6	1	1.1/	.82	4	5
58	1	- 2	10	1	4.45	2.73	8	9
29	i i	5	18	1	3.58	2.52	10	10
30		- 2	58	2	.59	.32	0	7
31		3 -	b	1	1.14	.48	3	3
35		3 -	4	2	.28	• 47	5	4
33		3 -	14	- 3	31	.00	4	4
34		3 -	14	2	1.20	.73	1	7
35		3 -	52	3	34	.00	3	3
36	1	3 -	58	2	• 40	.00	2	S
37		3 -	30	Э	-1.15	.13	8	7
38		• •	1	5	. / 4	.85	3	5
34		+ -	10	3	-1.36	• 0 0	10	10
40		+ -	20	S	.20	.05	2	1
41		4 -	22	3	39	.00	3	4
46	- Š	+ -	25	1	. /8	.51	1	1
43		4 -	20	4	.13	. 73	3	4

TABLE 10.NATURAL PERIODS FOR WATER YEAR 1963. THE CODE FOR -TYPE OF PERIOD-IS 1=STORM.2=PRECIPITATION.3=EVAPORATION,4=UNDEFINED. UNITS FOR P=E AND PG DATA ARE CM OF WATER DISTRIBUTED OVER THE BASIN

PERIOD	DATE	OF	START	IYPE OF	P-E	PG	LENG	TH OF
NUMBER	UF	PER	0019	PERIOD	(CM)	(CM)	PEI	COIN
	MUN	IH-L	YAY				P-E	P
1	10	- 0	1	4	79	.00	5	2
2	1	0 -	3	1	1.80	1.47	4	4
з	1	- 0	7	2	.52	.00	-4	4
4	1	0 -	11	3	-1.24	.10	5	4
5	1	• 0	10	1	2.24	1.96	4	6
6	1	0 -	20	3	92	.00	5	4
1	1		25	2	. 42	.00	2	2
в	1	- 0	27	3	99	.07	18	18
9	1	1 -	14	1	1.89	1.14	3	4
10	1	1 -	17	2	. 45	.23	3	2
11	1	1 -	20	3	69	.00	5	5
12	1	1 -	25	2	1.54	.36	9	10
13	1.	5 •	4	3	76	.00	12	12
14	1.	- 5	16	2	.01	. 40	4	3
15	13	2 -	20	3	52	.04	3	3
16	1.	2 -	23	2	1.00	. 48	в	8
17	1	2 -	31	3	41	.40	в	8
18		1 -	в	1	1.01	.72	4	4
19		1 -	15	2	.26	.02	4	3
20		1 -	16	1	1.07	.97	4	5
21		1 -	20	2	. 41	.00	2	5
22		1 -	22	з	30	.00	5	2
23		1 -	24	2	.30	.20	2	3
24		1 -	26	3	18	.00	3	2
25		1 -	24	1	1.12	2.80	3	4
26		5 -	1	3	-1.24	.35	9	8
27	1	5 -	10	2	1.27	1.10	11	15
28		5 -	51	3	80	.02	5	4
29		5 -	26	5	1.27	1.08	в	8
30		3 -	6	3	21	.00	3	3
31		3 -	4	2	. 15	.50	5	5
32		3 -	14	1	1.56	1.26	5	5
33		3 -	19	2	. 62	.07	6	6
34		3 -	25	3	46	• 40	5	4
35		3 -	30	1	1.01	• B 0	4	6
36		4 -	3	3	27	• 0 0	3	1
37		4 •	6	1	1.00	.29	5	3
38		4 -	8	3	15	.00	1	-0
39		4 -	9	1	1.20	.83	2	2
40		4 -	11	3	18	.05	3	5
41		4 -	14	1	3.04	.94	10	8
42		4 -	24	2	• 01	.00	2	3
43		4 -	26	1	1.85	1.26	3	4
6 4		4	20		- 7.1	110	2	

Year		Storm Pe	riods	J	Precipita Periods	tion 3	E	Evaporatio Periods	n	A11	Periods
	N	P-E	$\mathbf{P}_{\mathbf{G}}$	N	P-E	PG	N	P-E	PG	N	P-E*
1957	16	34.94	23.27	16	7.61	6.80	15	-5.86	1.06	47	36.69
1958	17	23.76	18.89	16	6.33	6.70	17	-9.15	1.76	50	20.94
1959	14	18.93	9.27	15	9.12	6.66	14	-6.72	1.91	43	21.33
1960	14	21. 52	13.65	18	11.18	4.95	18	-6.20	0.49	50	26.50
1961	12	21.79	13.36	18	10.41	7.12	20	-7.89	0.33	50	24.31
1962	14	29.19	18.47	17	7.89	3.84	10	-7.13	1.81	41	29,95
1963	12	19.39	14.44	14	9.43	4.44	16	-9.32	1.40	42	19.50

* Total P-E does not include undefined periods at start and end of season.



Figure 20. Seasonal accumulation of P-E for each type of natural period plotted against the total water accumulated during the season.

Synoptic patterns associated with the classes of natural periods: It has been pointed out previously in this paper and others (e.g., Rasmussen, 1963) that the large precipitation events occurring over the Upper Colorado River Basin are associated with well-developed slow-moving cyclones with 500 mb centers traversing over or just south of the basin. It is of interest to investigate the synoptic patterns associated with the precipitation and evaporation periods as well as the storm periods.

A large number of 500 mb and their corresponding surface maps were visually inspected in order to determine which synoptic parameters should be tested for variations between classes. Qualitatively, the storm periods are characterized by a strong cyclonic system west of the basin. The cyclone may or may not include a closed circulation aloft. The evaporation periods are characterized by an almost opposite circulation system dominated by a ridge aloft to the west of the basin and often this ridge is reflected at the surface by a well-developed high pressure center to the northwest of the basin. The smaller precipitation periods are generally characterized by quite variable conditions at 500 mb. Generally, the flow is almost due west with small perturbations traveling rapidly from west to east. The surface pressure systems are not nearly as intense as for the storm or evaporation cases and they move rapidly across the map. A striking feature of the storm and evaporation periods is the persistence of the 500 mb circulation pattern over days. This is not so apparent for the precipitation periods.

In order to provide some relevant statistics to the variations of synoptic patterns with respect to the natural period classes, two variables were chosen. First, the 500 mb wind direction over the basin was obtained visually from the Historical Daily Weather Map Series (U.S. Weather Bureau). A total of 992 separate daily values were obtained and the data grouped in 30[°] increments for each class of period. Figure 21 shows the percent frequency distribution of the grouped data for each type of period. One observes a trend from predominately southwest flow for the storm periods to northerly flow for the evaporation periods.

The second parameter tested was that of the occurrence or nonoccurrence of a surface high pressure center over the portion of the United States in the northwest quadrant of the compass centered on the basin. Again, the same map series was used and the data tabulated for the same 992 samples. Table 12 gives the results in terms of percent frequency of occurrence or non-occurrence for each natural period class. Again, the delineation between the natural period types is quite striking.

TABLE 12

Percent Frequency of Occurrence and Non-Occurrence cf a Surface High Pressure Center Northwest of the Basin

Type of Period	Occurrence of Cent	High Pressure er
	Yes	No
Storm	24	76
Precipitation	51	49
Evaporation	81	19

In summary, then, the storm situations are characteristically periods of persistent southwest flow over the basin with no strong high pressure area to the northwest of the basin. The precipitation periods are characterized by westerly to northwesterly flow aloft with rapidly moving disturbance imbedded in the general flow. The



Figure 21. Percent frequency distribution of the daily 500 mb wind direction over the basin for the three classes of natural periods.

evaporation periods are characterized by persistent northerly flow over the basin, a ridge to the west at 500 mb and a surface high pressure center to the northwest of the basin.

The Storm Periods

It was demonstrated in the preceding section that the storm periods are largely responsible for the seasonal water balance of the Upper Colorado River Basin. Several other questions can be asked with respect to the storm period results. What portion of the seasonal accumulation of water over the basin is due to the storm systems? Is it the number of events or the magnitude of individual events that determines the seasonal accumulation? Is it the daily intensity of the precipitation or the storm duration that determines storm yield?

Finally, assuming that under the meteorological conditions associated with storm periods the evaporation from the basin is negligible, what is the relationship between the storm precipitation determined from the water balance and that from the gauge data?

Table 13 lists the seasons in decreasing rank order with respect to the total seasonal yield of the storms along with the values for the total number of storm days, average length of storms, average yield, and the percent of the seasonal accumulation due to the storms. The last column lists the frequency of storm events each year that individually produced more than 3 cm of water.

The data presented in Table 13 quite pointedly answers the first two questions posed above. First, for the seven winters studied, the storms provide from 80 to 110 percent of the total seasonal accumulation of water over the basin. The average yield of the storm periods for the seven winters is 95 percent of the total water accumulated over the basin. Second, the number of storm events per year varies from 12 to 17 over the seven winters with little relationship between the number of events and the total storm

TABLE 13

Table of Statistics of the Storm Periods for the Seven Winter Seasons. The Years are Rank Ordered from the Highest Total Seasonal Storm Precipitation

Storm P-E (cm)	Year	No. of Storms	No. of Storm Days	Averagc Duration (Days)	Average Yield (cm)	Percent of Seasonal P-E	No. of Storms Yielding 3.0 cm or more
34.94	1957	16	81	5.1	2.18	95	4
29.19	1962	14	70	5.0	2.08	97	3
23.76	1958	17	58	3.4	1, 40	113	0
21.79	1961	12	47	3.9	1, 81	90	1
21, 52	1960	14	52	3.7	1.54	81	0
19.39	1963	12	48	4.0	1.60	99	1
18.93	1959	14	53	3.8	1.35	89	1

production. The yield of individual storms is better related to the total storm yield than the number of storm events. The average yield per storm is greatest for the two wettest years and least for the dryest year. Further, the frequency of very large storms, storms each yielding 3.00 cm of water or more, is much greater for the very wet years; without these three or four large storms, the seasonal storm yield and hence the seasonal accumulation of the two wettest winters would be of the same general magnitude as the other five years.

Figure 22 is designed to shed light on the question of whether the daily intensity or the storm duration determines the storm yield. This diagram shows the average duration of storms grouped with respect to storm yield. The result shows that the storm yield is largely a function of storm duration. This result amplifies the data given in the columns listing the total number of storm days and the average storm length for each of the water years in Table 13 above.

Figure 23 is a plot of the total sample of storm precipitation data computed from the atmospheric water balance against that derived from rain gauges. The correlation coefficient between the storm precipitation estimates is r = 0.8. The solid line denotes the line of perfect agreement. Eighty-two percent of the cases show the precipitation computed from the water balance to be greater than that determined from the gauge data. The dashed line is the linear regression fitted to the data, the functional expression for this line is: P (water balance) = $.7 + .9 P_G$. Thus, even for the case where the conditions are most favorable for equivalence between the water balance and precipitation gauge data, the precipitation gauge data is generally of lesser magnitude.



Figure 22. Average duration of storms classed according to yield.



Figure 23. Storm P_G plotted against storm P-E. Solid line is the line of perfect agreement, dashed line is the regression line fitted to the data.

The Evaporation Periods

The relationship between the wintertime evaporation occurring during periods of net evaporation over the basin and the wintertime accumulation of water over the basin was presented previously in this paper. Several additional aspects of the evaporation periods were studied. Some question can be raised concerning the validity of the very short evaporation periods, periods lasting a day or two, due to the possible errors inherent in the computation. Further, some of the periods exhibit a considerable amount of basin precipitation recorded in the gauge data. Because the sample size of evaporation periods is quite large, and in order to obtain the best possible computational times, the total sample of evaporation periods was reduced to include only those periods lasting more than two days and having a daily average of $P_{\rm G}$ of .01 cm or less. This amended sample of evaporation periods is used in this section.

The basin evaporation is a result of the interplay of many meteorological and hydrological variables of which one is the availability of water for evaporation. The wintertime climate of the Basin is typified by the season-long snow pack existing only in the high elevations. The occasional snows that cover the lower elevations do not last, in general, for more than a week or so following the storm. It is reasonable to assume that a considerable amount of this water is evaporated immediately following the storm period. It is apparent from the daily data presented in the section of this chapter that the evaporation periods generally follow the storm periods. The question was asked: What is the relationship between the total water evaporated or the daily rate of evaporation to the total water accumulated over various time increments preceding the evaporation period? No relationship was found to exist between these variables. What was determined was that the total water evaporated during an evaporation period could be accounted for by the

accumulation of water over a very few days just prior to the evaporation period. Seventy-six percent of the cases needed only four days or less preceding the evaporation period to accumulate the necessary evaporated water. Only four percent of the cases needed more than ten days and these almost exclusively occurred during March and April. This perhaps is a reflection of evaporation from the snow pack.

It is reasonable to expect the rate of evaporation from a soil surface to decrease as the time from the start of the evaporation period increases. This decrease in evaporation rate should be in response to the drying of the evaporating soil surface or perhaps in response to the change in character of the snow surface. Figure 25 shows the decay in average daily evaporation rate with time from the start of the evaporating period. The three lines delineate the decay rate for different portions of the winter season. Because of the relatively large number of short periods in the sample, the data were grouped with respect to time from start of period in order to obtain a similar sample size for each group. The average value is plotted at the class mark of the various groups. Because of the complicated nature of the evaporation process, wide variation with respect to evaporation rate occurs within each group, but the average values for each curve show a consistent change with time from the start of the period so the result was considered meaningful. A Few points of interest are apparent from the curves. First, the decay of evaporation with time is similar for early and middle winter with the exception that the evaporation rate on the first day of the period is almost a factor of 2 less for the colder portion of the season. Secondly, the evaporation rate does not decrease nearly as rapidly during the late winter. This perhaps can be explained by the occurrence of more evaporation coming from the wet surface of the high elevations.



Figure 24. Days prior to evaporation period needed to accumulate the quantity of water evaporated during the evaporation period.



Figure 25. Decay of evaporation rate with time from start of the evaporation period.



Figure 26. Seasonal trend of average daily evaporation rate. Bars indicate average monthly values. Circles indicate extreme values.

Finally, the seasonal trend of daily evaporation rate is shown in Figure 26. Here the bar graph indicates the mean of all average daily rates for evaporation periods with starting dates within each month. The crosses show the extreme average daily rate for each month. A similar trend is observed for both the monthly mean rate and the extremes. The evaporation rates vary by almost a factor of 2 over the season.

One of the factors determining the evaporation rate is the solar radiation received at the earth's surface. Daily values of solar radiation at Grand Junction. Colorado, are published in the Climatological Data - National Summary (U.S. Weather Bureau). Using this data one can determine the evaporation due to the solar heat source if one assumes that all of the radiation that is received, less that reflected, is used to evaporate water. This extreme value may be considered the "evaporative power" of the solar radiation. Table 14 gives the average daily values of radiation received at Grand Junction during the seven winter months of 1960 and also the extreme daily value for each month. These data were then reduced using albedo of 70 and 10 percent to typify the reflection from snow and bare soil conditions respectively. The evaporation power of the radiation was then computed assuming a heat of vaporization of 600 calories per gram of water evaporated. The results tabulated in Table 14 compared to the seasonal trend of evaporation rate shown in Figure 26 demonstrate that even on an "average" day with an albedo of 70 percent, the solar radiation is sufficient to explain the observed average evaporation. Similarly, on days where the meteorological conditions are such that a maximum possible solar radiation is approached the radiation can totally explain the extreme values shown in Figure 26. Under conditions where the albedo is less than 70 percent, as it undoubtedly is over the Colorado Basin for large portions of the winter season, only a fraction of the solar radiation

would be necessary to yield the observed evaporation. Other physical processes such as the conduction of heat from the atmosphere to the evaporating surface would enhance the evaporative process. The conclusion of this analysis is that the values of evaporation rate shown in Figure 26 are certainly plausible, particularly since they occur under meteorological conditions conducive to clear skies and high solar radiation amounts. TABLE 14. Computation of evaporation at Grand Junction, Colorado, assuming the total solar radiation received, corrected for albedos of 70 percent (snow surface) and 10 percent (soil surface), is used for evaporation. The evaporation is computed using the average daily insolation and the maximum observed during the winter months of 1960.

	AVERAGE	DAILY VAL	UE	MAXIMUM DAILY VALUE			
MONTH	Solar Radiation	Evaporation (cm/day)		Solar Radiation	Evaporation (cm/day)		
	(ly/day)	A1bedo 70%	A1bedo 10%	(ly/day)	Albedo 70%	Albedo 10%	
October	353	.17	. 53	489	. 25	. 73	
November	263	. 12	. 39	336	.17	. 50	
December	235	.11	. 35	275	.14	. 41	
January	220	. 11	. 33	318	. 16	. 48	
February	309	.16	. 46	510	. 25	. 76	
March	480	24	. 72	624	. 31	. 94	
April	576	. 29	. 86	772	. 38	1.15	
CHAPTER V

THE HYDROLOGIC BALANCE OF THE JPPER COLORADO RIVER BASIN

As presented in Chapter II, the hydrologic balance of a river basin can be written

 $P - E = R_{o}^{*} + \Delta W$

where R_0^* is the runoff corrected for diversions from the basin and ΔW is the change in storage of ground water over the basin. The correlation between (P-E) Oct. -April and (Ro)Oct. -Sept. was r = .64; this, of course, assumes \triangle w to be unimportant. The problem of estimating the change in storage or carryover of water mass stored in the subsurface soil moisture is particularly hard to estimate. The runoff from the Upper Colorado is largely derived from the melt of snow in the high elevations of the headwaters and, thus, there is a lag between the deposition and resulting runoff. As a first approximation to the determination of total runoff from a single winter season accumulation, one may assume some set lag time between the accumulation and runoff and, therefore, minimize the magnitude of the carryover. Figure 27 shows the monthly regime of runoff from the Upper Colorado Basin measured at Lee's Ferry, Arizona, for the water years 1957, 1958, and 1959. It is apparent that the maximum runoff occurs during the spring and early summer months and it tapers off during the winter. In order to test the relationship between winter accumulation of water over the basin and the resulting runoff, it was decided to compare the winter precipitation to runoff beginning at April 1 of the year of record and ending March 31 of the year following. It is assumed that this lag process



Figure 27. Solid line is the monthly course of runoff measured at Lee's Ferry, Arizona, for the water years 1957-1959. The hatched area is the amount of runoff assumed to be due to the October, 1956, through April, 1957, deposition.

accounts for a large portion of the carryover from year to year. No test of this technique is available and the number of years is too small to statistically derive the best relationship between winter accumulation and runoff evaluated using various lag times.

Figure 28 shows the wintertime P-E plotted against April through March runoff for the seven year sample. The correlation between the parameters if r = .84, a considerable improvement from the case where Δw was neglected. The regression line fitted to the data is entered as the solid line. The functional form for the linear relationship is

R^{*}_{o April-March} = -3.3 + .3 (P-E)_{Oct-April}

The runoff is roughly one-fifth of the winter accumulation; hence, four-fifths of the winter accumulation must be evaporated during the summer season.

Note should be taken of the large deviation in the plot for water year, 1958. The April to March runoff for 1958 could include considerable carryover from the very wet year, 1957, and thus an adjustment yielding a much better relationship perhaps is merited. Note should be taken that the maximum runoff for the period April, 1958, to March, 1959, occurred in May, 1958, a deviation from the average pattern which shows a maximum in June. This perhaps is a reflection of the carryover of soil moisture from the preceding year allowing the summer peak discharge to occur earlier.

The point of this hydrologic analysis is that the annual discharge from the Upper Colorado is largely described by the wintertime atmospheric water balance. Further, the result suggests a scheme for forecasting the annual runoff from the Colorado River Basin. It is difficult to forecast runoff from large areas using standard precipitation and snow course data (Ford 1959). The attractive feature of the atmospheric water balance technique as displayed



Figure 28. Seasonal (October through April) P-E computed from the atmospheric winter balance plotted against the April through March runoff at Lee's Ferry. The solid line is the linear regression fitted to the data.

here is that the day-by-day accumulation is monitored and the effect of extended periods of dryness as well as precipitation are accounted for.

CHAPTER VI

CONCLUSION

The many specific results of the atmospheric water balance of the Upper Colorado River Basin were stated individually in the text and will not be reiterated here. A general description of the results with respect to the questions posed in the Introduction will be given.

The hydrological balance of the Upper Colorado River Basin for the winter seasons of 1957 through 1963 was determined using the atmospheric water balance approach. The correlation between the winter accumulation of water and the April through March runoff was r = .84. The linear relationship between the values was:

(R^{*}_o)_{April-March} = -3.3 +.3 (P-E)_{Oct-April}

This result is based on a gross simplification of the carryover of stored water from year to year, but the result is encouraging considering the crude approximation. The relationship between the winter atmospheric water balance and the annual river discharge suggests a technique for forecasting the annual flow of the river.

The seasonal accumulation of water over the basin was shown to be largely determined by periods of net evaporation as well as storm periods. Periods of small net precipitation, on the other hand, do not explain much of the seasonal variation of accumulated water.

The general synoptic patterns associated with periods of precipitation and evaporation were found to be quite different. The parameters chosen to delineate this difference were the wind direction at 500 mb over the basin and the occurrence or non-occurrence of a surface high pressure center to the northwest of the basin. Several features of the evaporation periods were determined. The results include a description of the decay in time of the evaporation rate during periods of basin evaporation and the seasonal variation of daily evaporation rate.

Finally, from the large sample of daily data used in this study, it was found that the basin precipitation as determined from rain gauges is about 50 percent less than that obtained from the atmospheric water balance. A large portion of this deficit is due to the lack of sampling over the high elevation regions of the basin.

In spite of the many computational problems inherent in the evaluation of the atmospheric water balance, a meaningful computation can be performed for a $2 \times 10^5 \text{km}^2$ area over periods ranging from days to seasons. This method is particularly applicable to arid regions with little historical hydrologic data but where the need for knowledge is necessary in the face of pressing water resource problems.

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APPENDIX A

THE OBJECTIVE ANALYSIS SCHEME

The data handling requirements of a computational procedure such as described in this paper are massive. The researcher could not hope to begin to draw by hand the necessary maps of all the variables at all the levels for all the days covered in this work. To do this task a digital computer was coded to objectively analyze the data and interpolate the data fields to the grid shown in Figure ⁵ in the text. The method employed is common in meteorological analysis and consists of fitting a quadratic surface to each parameter at each level and then taking the value of the surface at the grid point as the interpolated variable value. Variations of such a scheme have been published in the literature, for example Panofsky (1949), Gilchrist and Cressman (1954) and Baer and Kamm (1965). Other methods of objective analysis are available (e.g., Cressman, 1959), these methods are usually based upon some weighting factor technique and are particularly adaptable to areas with few and widely scattered observations. The Colorado River Basin is located in such a way that there is an abundance of observation locations in and entirely around the area, thus the quadratic surface fitting scheme was chosen.

Let us signify data points with the subscript d and the grid points with the subscript g. The distances between a grid point and a data point may be written

$$x_{d} = (\lambda_{d} - \lambda_{g}) (\cos \frac{\theta_{g} + \theta_{d}}{2})$$
$$y_{d} = \theta_{d} - \theta_{g}$$

where λ is degrees longitude, θ is degrees latitude, x is distance eastward and y is distance northward. It is assumed that any variable ξ , on any pressure surface can be expressed by a quadratic surface

$$\xi(x,y) = a_0 + a_1x + a_2x^2 + a_3xy + a_4y^2 + a_5y$$

Clearly, one would need six data points to evaluate the coefficients $a_0 \ldots a_5$. More than six data points are usually available, however, so the "best fit" of the surface to the data over some influence region was determined by the method of least squares. The influence region was fixed by the particular distribution of observation locations used in this study and was defined as that region within a radius of 6.5 degrees latitude of the grid point. All observations outside this influence region were disregarded for the evaluation of the polynomial at that grid point.

By the method of least squares we define a deviation

 $D = \sum_{d} \left[\xi_{d} - (a_{0} + a_{1}x + a_{2}x^{2} + a_{3}xy + a_{4}y^{2} + a_{5}y) \right]^{2}$ which is required to be a minimum, hence $\frac{\delta D}{\delta a_{0}}$, $\frac{\delta D}{\delta a_{1}}$, $\frac{\delta D}{\delta a_{5}}$ are all zero. This operation yields the six normal equations which are then solved for the coefficient a_{0} ; a_{0} is the value at x = 0, which is the location of the grid point. The method of solving the six normal equations follows that of Crout (1941).

For each observation period 315 separate polynomials were fitted to the data. These computations plus the evaluation of the atmospheric water balance required six seconds per observation period on the CDC 6600 computer using a program coded in Fortran language.

APPENDIX B

LIST OF SYMBOLS

- A = Area on a horizontal surface
- $C_n = Normal wind component$
- D = Wind direction
- e = Partial pressure of water vapor
- E = Evaporation Rule
- F_{L} = Divergence of flux of liquid water or ice
- g = Acceleration of gravity
- H = Height of pressure surfaces
- i = Index enumerating opperations on a pressure surface
- j = Index enumerating opperations in the vertical
- l = Boundary length
- L = Man-made depletions of water from a river basin
- m = Limits of summation

M = Mass

- n = Limits of summation
- p = Pressure
- P = Precipitation rule

 P_G = Precipitation rule obtained from gauge data

- q = Specific humidity
- r = Ratio of mass of water to mass of moist air
- $R_0 = Runoff$
- R^*_{o} = Runoff corrected for depletions
- s = Relative humidity

t = Time

- T = Temperature
- V = Wind speed

- IV = Wind velocity vector
- w = Vertical motion
- W = Water mass stored in the ground and on surface
- W = Precipitable water mass
- x, y, p = Coordinate system with p as vertical coordinate

x,y,z = Cartesian coordinate system

 Z_s = Elevation of topography

- Σ = Ratio of molecular weights of water vapor to dry air
- ξ = Generalized variable

ϕ = Latitude

- λ = Longitude
- ρ = Density
- σ = Area increment of a vertical section
- ω = Total change of pressure with time
- ∇ = Gradient operator

