MASTER'S REPORT

THE DESIGN OF STABLE CHANNELS IN ERODIBLE MATERIAL

Submitted by

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WE HEREBY RECOMMEND THAT THE REPORT PREPARED UNDER OUR SUPERVISION BY KING YU

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INTRODUCTION

Many important problems must be solved by engineers when designing channels in erodible material and unfortunately their complete solution requires more knowledge of the subject than now exists. The primary purpose of the designing work is to obtain a stable channel which will have the least initial cost and least maintainance cost. For a channel to be stable, the hydraulic features and the characteristics of the bed material and the material in suspension must have a combined effect that will cause neither silting nor scouring. To this end, British Engineers have done very intensive study in connection with large irrigation canals in India. Their solutions, however, have not applied so well in the regions other than those where the data were obtained. A generalized analysis of stable channels is reported in this manuscript.

The problem

In what ways may the problems of stable channels in erodible material be solved more satisfactorily? This question may be subdivided into several items as follows:

1. What have previous workers done in this field?

2. What are the important factors involved, and which of these factors have previous investigators neglected?

3. What are the proper methods of attacking this problem?

4. Can the shape of a stable channel cross-section be expressed mathematically?

5. Do the available data verify the results of theoretical and dimensional analyses?

6. What conclusions and recommendations may be made as a result of this study?

It is the purpose of this manuscript to summarize the information now available on the subject of stable channels and analyze the problem from both the theoretical and dimensional point of view. To the extent that the limited available data will permit, these developments are tested experimentally.

Chapter II

REVIEW OF LITERATURE

Although in this chapter the citation of the literature are essentially in chronological order, each author developed his own series of formulas for design of stable channels and published them at different times as the research progressed. Hence the chronological presentation is based upon the time of the first important contribution of a given author. The English system of units is used in all equations unless otherwise stated.

As will be noticed, pioneering studies of stable channels have been made by British engineers in India although some have been made in Egypt. Interest in this problem in the United States has developed only within the last thirty years.

Kennedy (14) using the data of the Upper Bari Doab Canal in India, empirically derived the following equation for stable channels in 1895:

$$V_c = 0.84 \, m \, y^{0.64} \tag{2.1}$$

in which V_c is the critical velocity which will just keep the channel free from either silting or scouring, y is the water depth of the channel exclusive of side slopes (most of the channels were rectangular in cross-section), and m is a coefficient. A standard for the coefficient m of 1.0 for sandy silt was adopted. Coarser sand had values of 1.1 to 1.2 and finer sand 0.8 to 0.9.

Kennedy determined the slopes of the channels by assuming a value of N for Kutter's equation which is in turn applied to Chezy's equation. He suggested N be 0.02 for large canals and 0.0225 for small canals.

Woods (28) in 1917 proposed the use of definite ratios of depth to width, based on an analysis of data from the Lower Chenab Canal System. In 1927, he published his general equations:

$$\log D = 0.434 \log W_a \qquad (2.2)$$

$$V_c = \log W_a D = 1.434 \log W_a \qquad (2.3)$$

$$S = 1 / \log Q^2 = 1000 s \qquad (2.4)$$

in which D is the average depth of water, W_a is the mean width of water, V_c is the non-silting, non-scouring mean velocity, Q is the rate of discharge or $W_a D V_c$, and S is the water surface fall in 1000 feet distance or 1,000 s.

Woods believed that the sediment carrying capacity of a shallow channel is greater than for a deep channel and depends on $\frac{(v_s - v_b)}{D}$, where v_s is the surface velocity and v_b is the bottom velocity.

In 1919, Lindley (23) published the following equations developed from 786 observations made on 2700 miles of channels in Jhang and Lyallpur, India:

$$V_c = 0.95\ y^{0.57} \tag{2.5}$$

$$V_c = 0.59\ y^{0.355} \tag{2.6}$$

$$B = 3.8\ y^{1.61} \tag{2.7}$$

in which B is the bed width of channel in feet. Kutter's equation was used to compute the velocity of flow. Believing that the ratio of bed width to depth plays an important part in determining stable sections, he put forward the theory that "the dimensions: width, depth, and gradient, of a channel to carry a given supply loaded with a given silt charge, were all fixed by nature."

Molesworth and Yenidunia (20:129) developed the following general equation in 1922 from a careful examination of a large number of stable Egyptian canals.

$$y = (9060\ s + 0.725\ B) \tag{2.8}$$

In 1923 Buckley (3) developed a modification of Eq. (2.8) for channels of depths less than 5.26 feet (1.6 meters).

$$y = \frac{0.0025\ (100000\ s + 8)^2\ B}{1.62} \tag{2.9}$$

As reported by Lacey (16), Buckley later derived an equation from measurements of the Nile at Belieda:

$$V = m\ R^{0.85}\ S^{0.72} \tag{2.10}$$

in which R is the hydraulic radius.

As reported by Lacey (16), Phillips also developed an equation for the Nile at Beleida.

$$V = 2.08 \ (RS) \qquad (2.11)$$

From 1926 to 1946 Griffith (8, 9, 10, 11) established his basic law and claimed that changes in the cross-section of a channel, which resulted from changes in the prevailing hydraulic conditions, could be mathematically calculated by reference to this basic law. His work was based on the Kutter and Chezy equations and did not use the existing canal data. In his latest paper (8), published in 1946, he used the data of the Missouri River to check his equations. His work can be summarized as follows:

1. The basic law of equilibrium of the bed at any point in the cross-section is:

$$v = m \ y^{(0.5 + 3.33N)} \qquad (2.12)$$

in which v is the mean velocity in a vertical line through the point under consideration, y is the water depth at this point, m is the general sediment factor, and N is the Kutter coefficient of roughness.

2. The general equation for the equilibrium of a cross-section is

$$V = k \ m \ D^{(0.5 + 3.33N)} \qquad (2.13)$$

in which V is the mean velocity of the cross-section under consideration, D is the mean depth of the cross-section, and k is the ratio of V/v_a where v_a is the mean velocity in the vertical of mean depth. The following values of k were given:

For rectangular cross-section	$k = 1.0$
For triangular cross-section	$k = 1.13$
For semi-elliptical cross-section	$k = 1.055$

3. The general equation for the equilibrium of cross-sections is of similar type:

$$V = m\, D^{(0.5 + 3.33N)} \tag{2.14}$$

4. The equation relating the sediment factor to the sediment concentration is:

$$c = Z\, m = \frac{Z\, v}{D^{(0.5 + 3.33N)}} = \frac{Z\, V}{D^{(0.5 + 3.33N)}} \tag{2.15}$$

in which c is the sediment concentration of bed load in parts per 10,000 by weight of water, and Z is a sediment coefficient (for fine sand $Z = 7.5$).

5. The equations for regime sections:-

$$R = \frac{0.35\, Q^{0.314}}{(1000\, s)^{0.22}} = \frac{Q^{0.314}}{13.06\, s^{0.22}} \tag{2.16}$$

$$P = \frac{48.75\, Nm\, Q^{0.477}}{s^{0.133}} \tag{2.17}$$

$$m = \frac{V}{D^{0.5 + 3.33N}} = 38.4\, s^{0.47}\, Q^{0.058} \tag{2.18}$$

$$G = 1.8\, s^{0.47}\, Q^{1.058} \tag{2.19}$$

in which R is the hydraulic radius, P is the wetted perimeter, Nm is Manning's coefficient of roughness, and G is the rate of discharge of bed load in pounds per second.

During the period 1929 to 1946 Lacey (16, 17, 18, 19) published several important papers based on data from major irrigation canals in India.

His final equations may be summarized as follows:

1. The relation between velocity and hydraulic radius:

$$V = 1.151 \sqrt{f R} \quad \text{or} \quad f = 0.75 V^2/R \qquad (2.20)$$

in which f is the silt factor.

2. The relation between discharge and wetted perimeter:

$$P = 2.668 \sqrt{Q} \quad \text{or} \quad P/R = 7.11 V \qquad (2.21)$$

in which P/R is a "shape factor".

3. The flow equation:

$$V = 16 (R^{1/2} s)^{1/3} \qquad (2.22)$$

Substituting Eq. (2.20) into Eq.(2.22):

$$V = \frac{1.3458}{N_a} R^{3/4} s^{1/2} \qquad (2.23)$$

in which

$$N_a = 0.0225 f^{1/4} \qquad (2.24)$$

where N_a is the absolute rugosity, defined by the gradation of the sediment and as a function of average sediment diameter.

4. Relation of silt factor to bed material diameter:

$$f = 1.76\sqrt{d} \qquad (2.25)$$

in which d is the mean diameter of bed material in millimeters.

In Lacey's recent interpretation of his theory (16), he re-emphasized his belief that his equations are applicable to all active channels in alluvium in general, irrespective of the precise degree of scouring or silting, or the variation in sediment charge. He rearranged Eq. (2.22) to obtain

$$V\ S = 1.60\ (R^{1/2}\ S)^{4/3} \qquad (2.26)$$

in which $S = 1000\ s$ where s is the slope of the water surface. Lacey called Eq. (2.26) the "normal alluvial equation" and pointed out two significant parameters in this equation:

1st parameter, (VS) - directly proportional to terminal settling velocity of sediment particles and espitomizing the forces tending to restore the particles to the bed.

2nd parameter, $(R^{1/2}S)$ - a function of sediment size, sediment density, and sediment discharge and espitomizing the forces tending to propel them forward.

The upper limit of Eq. (2.26) will be reached when the critical velocity occurs $v = \sqrt{gD} = \sqrt{gR}$
From Eq. (2.26), he obtained:

$$V\ S = g^{2}/4.096 = 253 \qquad (2.27)$$

$$R^{1/2}\ S = g^{3/2}/4.096 = 43.5 \qquad (2.28)$$

The lower limit of (V S) is due to the cohesion of very fine sediment.

In connection with Lacey's work, Bottomley (2) equated Manning's equation to Lacey's Eq. (2.22) to find an expression of Manning's coefficient of roughness in terms of the observed slope:

$$N_m = 0.00928\ S_m{}^{1/6} \tag{2.29}$$

in which $S_m = 1{,}000{,}000\ s$

For N_a, Lacey re-arranged Eq. (2.22) as:

$$V = \left[(16)^{9/8} / (Vs)^{1/8}\right] R^{3/4}\ s^{1/2}$$

which combines with Eq. (2.25) to give:

$$N_a = 0.0596\ (V\ s)^{1/8}$$

or

$$N_a = 0.025.\ (V\ S)^{1/8} \tag{2.30}$$

As a summary of his work, Lacey stated the following conclusions:

1. A stable channel is one that is active and neither silting nor scouring.

2. A regime channel is a stable channel transporting a regime sediment discharge.

3. A regime sediment discharge is the minimum transported sediment load consistent with a fully active bed.

4. For every regime channel there is, for a given bed sediment grade, a fixed value for the product of the mean velocity and the slope.

5. The product (V S) is in all regime channels a criterion of the silt grade, and is proportional to the terminal velocity of the silt particles. This product is at all times an indication of the order of bed material.

6. The normal alluvial equation takes the form:

$$(V\ S) = 1.60\ (R^{1/2}\ S)^{4/3} \tag{2.26}$$

It may be re-arranged as:

$$V = 1.42\ (V\ S)^{1/4}\ R^{1/2} \tag{2.31}$$

which is the equation for the velocity of bed load propulsion. A working value of (V S) may be assigned to any channel.

7. All active channels in alluvium, free from "shock", whether silting or scouring or subject to an excess silt charge, conform with the normal equation.

8. The degree of silting or scouring, and the intensity of the sediment discharge, are implicit in the values of V, R, and S which a normal channel adopts when these values are associated with actual grade of sediment exposed on the bed.

9. If any active channel is subjected to "shock" by the presence of major irregularities of the banks or bed, the effective hydraulic mean depth is reduced, the slope increased, the velocity diminished, and the depth increased. There is no change in the bed sediment grade, or in the value of (V S) previously recorded.

10. As to the shape of the channel, it has been found that the wetted perimeter gives as good a correlation as the width. And the wetted perimeters of similar alluvial channels vary as the square roots of the discharge. (16:46-7)

Chatley (3, 4) published two series of articles in London in 1938. The general topic of the first series was: "River flow problems" and the second series, "River control problems." He based his developments on Chezy's equation, modifying it according to the Blasius relation between boundary resistance and the Reynold number.

$$V = \frac{1}{N_1}\ R^{0.71}\ S^{0.57} \tag{2.32}$$

in which N_1 is an arbitrary coefficient of roughness, involving the shape factor f' of the bed material, the density of fluid ρ, the coefficient of rubbing friction μ', and the kinematic viscosity of the fluid ν. In general, N_1 is smaller than Kutter's N. He compared Eq. (2.32) with Manning's equation:

$$V = \frac{1}{N_m} R^{2/3} s^{1/2}$$

and Forchheimer's equation:

$$V = \frac{1}{N} R^{0.7} s^{0.5}$$

The above equations, including Eq. (2.32), are all in metric units.

Chatley analyzed the shape of the channel by three separate methods. These were Chezy's equation, Manning's equation, and Eq. (2.32). His third equation gave

$$W = N_1 \left(\frac{\gamma}{T}\right)^{1.71} s^{1.14} Q \qquad (2.33)$$

in which γ is the specific weight of fluid, W is the width of the water surface, and T is the tractive force per unit area along the river bed. His assumptions were: (a) Eq. (2.32) holds true, (b) DuBoy's idea of constant tractive resistance at equilibrium is true, i.e. $T = \gamma Ds$, (c) The slope of the channel bed equals the hydraulic gradient. (d) Discharge Q is determined by drainage area and runoff factors. (e) The hydraulic radius R equals the mean depth D. (f) The soil structure is uniform.

In 1943, King (1,15) developed a series of equations from the data of canals in Punjab and Sind, India. In all equations he used the effective depth D and the water surface width W instead of the hydraulic radius R and the wetted perimeter P. He pointed out:

> The friction coefficient, which in pipes is the actual coefficient at any point, is in the case of channels an average coefficient, or alternatively may be considered as a maximum value distributed over an average perimeter less than the wetted perimeter and greater than the bed width. This average perimeter is assumed to equal the surface width W. (15:52)

Hence, he used W/D as the "hydraulic shape" and V^2/D as the "turbulence criterion" in comparison with Lacey's P/R and V^2/R.

King's final equations for designing canals are:

1. The regime check equation:

$$V = 80\, W^{1/7} (Ds)^{4/7} = \frac{3}{2} W^{1/7} (DS)^{4/7} \tag{2.34}$$

in which W is the width of water surface, and D is the effective depth or $\frac{A}{W}$.

2. The channel dimensions in terms of S and Q:

$$V = S^{1/3}\, Q^{3/14} \tag{2.35}$$

$$D = \frac{3}{7}\, Q^{5/21}\, S^{-4/9} \tag{2.36}$$

$$W = \frac{7}{3}\, S^{1/9}\, Q^{23/42} \tag{2.37}$$

3. The channel dimensions in terms of f and Q:

$$V = \frac{7}{9} f^{3/10} Q^{11/70} \tag{2.38}$$

$$D = \frac{3}{5} Q^{11/35} f^{-2/5} \tag{2.39}$$

$$W = \frac{15}{7} f^{1/10} Q^{37/70} \tag{2.40}$$

in which f corresponds to Lacey's "silt factor" except that King used $f = V^2/D$.

4. Slope in terms of f and Q:

$$S = \frac{7}{15} f^{9/10} Q^{-6/35} \tag{2.41}$$

5. The turbulence equation:

$$f = \frac{11}{4} d^{10/21} Q^{-1/21} \tag{2.42}$$

in which d is the mean diameter of sediment.

6. Dimensionless equations:

$$\underline{F}^{2}/S = 2.237\,\underline{R}^{1/3} \tag{2.43}$$

$$\frac{W}{D} = 0.124\,\underline{R}^{1/3} \tag{2.44}$$

$$\frac{\underline{F}^2}{S} = 3.763\,\underline{R}_W^{1/4} \tag{2.45}$$

$$\underline{F}^2 = 2.77\, d^{10/21} Q^{1/10} \tag{2.46}$$

where $\underline{F}$ is the Froude number or $\frac{V}{\sqrt{gD}}$, $\underline{R}$ is the Reynolds number or $\frac{VD}{\nu}$, and $\underline{R}_W$ is the Reynolds number with W as a length parameter or $\frac{VW}{\nu}$

In 1940 Straub (25) derived the equations for wide channels where side effects are negligible and R may be considered equal to D. He based his work on the following two equations:

DuBoy's equation: $G = X \frac{T}{\gamma} \left(\frac{T}{\gamma} - \frac{Tc}{\gamma} \right)$

Stickler and Gilbert's equation: $V = K\ D^{2/3}\ s^{1/2}$

in which G is the rate of bed load discharge per unit width, Tc is the critical unit tractive force at which bed movement begins, and X is the sediment parameter, an experimental coefficient. His equation is:

$$G = X \frac{s^{1.4}}{K^{1.2}} q^{3/5} \left[q^{3/5} - q_c^{\ 3/5} \right] \qquad (2.47)$$

where $q = DV$ is the discharge per unit width of channel and q_c is the discharge under critical conditions. In 1946 he (26) applied the principle of Eq. (2.47) to the suspended sediment discharge of the Missouri River past Kansas City as:

$$G = (30.4)\ (10^{-11})\ Q^{2.16} \qquad (2.48)$$

in which G is the rate of suspended sediment discharge in tons per second, and Q is the rate of water discharge in thousands of cubic feet per second.

A further equation paralleling Eq. (2.47) for the case of constant discharge was derived:

$$G = X\ q^{2/3}\ K^{1/4}\ V^{7/3}\ (V^{7/3} - Vc^{7/3}) \qquad (2.49)$$

Under equilibrium conditions, the average amount of sediment transported per unit volume of water must be the same at all sections of the canal, i.e. $\frac{G}{q} = K'$ (a constant). Hence, when $Vc \rightarrow 0$

$$K' = X \frac{1}{q^{5/3} K^4} V^{14/3}$$

Because

$$q = DV$$

$$V = \frac{(K')^{1/3} K^{4/3}}{X^{1/3}} D^{5/9}$$

or

$$V = m D^{0.56} \qquad (2.50)$$

in which m is a coefficient of the Kennedy-type equation and can be computed from a number of parameters defining the sediment and hydraulic characteristics of the stream. In other words, the value of m is a function of the sediment transportation characteristics, the sediment load, the water discharge, and the roughness of the channel.

Although Lane (20) did not develop any equations, an important and clear discussion of the principles involved in the design of stable channels in erodible material was brought out in his paper in connection with the design of the All-American Canal in 1937. He summarized the factors affecting the stable channel shape as follows:

> (a) Hydraulic factors (slope, roughness, hydraulic radius or depth, mean velocity, velocity distribution, and temperature); (b) channel shape (width, depth, and side slopes); (c) nature of material transported (size, shape, specific gravity, dispersion, quantity, and bank and sub-grade material); and, (d) miscellaneous (alignment, uniformity of flow, and aging). (20:131)

In 1947 Fai (7) published his study of stable channels in the Chinese Journal, Hydraulic Engineering. He derived an equation for the shape of stable channels in cohesive erodible material mathematically under the main assumption that the resisting force of the particles is constant along the circumference of the channel cross-section.

Chapter III

GENERAL ANALYSIS OF THE PROBLEM

Summary and analysis of previous work

The pioneering work on stable channel flow was started by Kennedy. Since his initial work was done, the same type of equation has been proposed by various investigators in a large number of special applications. In general, it should be noted, however, that such an equation ($V = my^n$) gives an expression only for the critical velocity, indicating that it increases with the depth. Kennedy gave a limiting critical velocity which corresponds to depths of about 10 feet. In fact, this limiting critical velocity depends largely upon the material which composes the bank and bed of the channel.

Woods, Lindley, and the Egyptian investigators introduced the relationship between velocity, bed width, hydraulic radius, and slope in addition to Kennedy's type of relation. They still gave no suggestion, however, that these relations might be influenced by the quantity or the quality of the sediment.

The most elaborate work in this field was Lacey's, although King's work was directly in parallel.

While both of them gave the relationship between channel dimensions and velocity and introduced the effect of the size of sediment, neither of them considered the quantity of sediment transported. Lacey used the wetted perimeter P, and the hydraulic radius R instead of the water depth y of the Kennedy-type equations. King strongly opposed Lacey. He pointed out that the idea of P/R measuring "hydraulic shape" was at variance with the real canal shape where the sides are quite different from the bed in nature as well as in appearance. He cited that introducing R originated from pipe flow studies where $\frac{A}{P} = R = \frac{r}{2}$. For the reason mentioned in Chapter II, he used water surface width W and $D = \frac{A}{W}$ as effective depth of water.

Because Lacey and King gave various equations which are helpful in the design of stable channels, a comparison of their work is given:

1. Similarity of "regime test" equation:

From Lacey's Eq. (2.22)

$$V = 16\ (R^2 s)^{1/3}$$

or

$$f = \text{const.}\ \ V^2/R = \text{Const.}\ \ (R^{1/2} s)^{2/3}$$

From King's Eq. (2.34)

$$V = 80\ W^{1/7}\ (Ds)^{4/7}$$

If R in Lacey's equation is replaced by D

$$V = \text{Const. } f^{2/9}\, D^{2/3}\, s^{1/3}$$

$$= \text{Const. } (W/D)^{1/7}\, D^{5/7}\, s^{4/7}$$

which is in almost the same form as King's.

2. Similarity of "f":

From Lacey's Eq. (2.22)

$$f = \text{Const. } \frac{V^2}{R} = \text{Const. } (R^{1/2}\, s)^{2/3}$$

From King's Eq. (2.34)

$$f = \frac{V^2}{D} = \text{Const. } W^{2/7}\, D^{1/7}\, s^{8/7}$$

3. Similarity of roughness and turbulence criteria

From Lacey's Eq. (2.23), and Eq. (2.24)

$$V = \frac{\text{Const.}}{f^{1/4}}\, R^{3/4} s^{1/2}$$

From King's Eq. (2.43)

$$\frac{V^2}{S} = 2.237\, \underline{R}^{1/3}$$

or

$$\frac{V^2}{D} = \text{const. } S * (VD)^{1/3}$$

or

$$V = \text{const. } f^{1/6}\, D^{7/6}\, S^{1/2}$$

4. Similarity of slope equations: Lacey transferred his equation to the form of the Punjab Research Institute equation and obtained the following form: (17:24)

Lacey's equation

$$S = f^{5/3}/1844.3\, Q^{1/6}$$

King's Eq. (2.41)

$$S = \frac{7}{15}\, f^{9/10}/Q^{6/35}$$

The f values calculated from the above equation of Lacey's were not consistent with the f values calculated from Eq. (2.22). The former varied as a power of the later around 0.5 to 0.6. This variation was removed by King's equations because $\frac{V^2}{D}$ is comparable to f.

Because King's equations were developed from a more fundamental method of analysis, his equations are probably more reliable for application to designing stable channels. The primary defect of King's work, however, is that he developed such a great number of different types of equations.

One should keep in mind the mutually dependent nature of those factors which control the stable conditions and understand the limitations of the equations which he uses.

All of the above mentioned investigators developed their equations from the data of existing canals. The following ones derived their equations mathematically.

Basing his work on the Kutter and Chezy equations, Griffith started with a basic law which was initiated by himself. He developed the velocity equation as a type similar to Kennedy's but the variation of m and n followed a definite law. Griffith also correlated the concentration of bed load with the coefficient m and the rate of discharge of bed load with the slope s and water discharge Q. Although some of these relations may be questionable, he nevertheless was approaching the problem in the advanced direction of research in this field.

Chatley and Straub both started from DuBoy's equation. Chatley introduced the Blasius relation between tractive force and the effect of Reynolds number, while Straub used the result of Strickler and Gilbert's studies. Equations of both authors are for wide channels, but Straub considered only unit width of channel while Chatley gave the relation between width W, slope s, and discharge Q. Because for wide channels the hydraulic radius is practically equal to the average depth, Chatley's width equation was unnecessary. Furthermore, the significance of the channel width depends upon the characteristics of the channel material--in some cases the width is not important while in other cases it is quite important. In the wide channels specifically studied by Chatley, the width of channel is comparatively unimportant.

Straub's work was more progressive. He is the only one among all the authors to put the concentration of the sediment into the equation. Unfortunately, however, he simply stated that the coefficient m is a function of the sediment transportation characteristics, the sediment load, the water discharge, and the roughness of the channel--no suggestion was made as to how to determine this complex function.

Lane made a detailed analysis of this problem and stimulated interest in further studies in this field.

Dimensional analysis

To make a dimensional analysis of this problem, it is necessary to study the various factors which control the channel flow. These factors may be divided into three groups:

A. Flow and channel characteristics:

1. V mean velocity of the flow
2. s hydraulic gradient of the flow
3. W water surface width
4. D average depth of channel, $D = \frac{A}{W}$

B. Fluid characteristics:

1. ρ density of the fluid
2. $\Delta\gamma$ interface difference of specific weight
3. μ viscosity of fluid

C. Sediment characteristics:

1. d' mean diameter of sediment transported
2. σ' standard deviation of sizes of sediment transported
3. d mean diameter of bed material
4. σ standard deviation of sizes of bed

material

5. ρ_s density of sediment

6. G rate of discharge of sediment

7. f' shape factor of sediment

8. f'_c a factor of cohesion of sediment

Other variables which are not listed in the foregoing, but mentioned by Lane (20) in his work on the design of the All American Canal, are roughness, velocity distribution, temperature, and channel aging. These are not included for the following reasons:

1. Roughness may be considered as a result of the combined effect of the size of bed material and bed load, and the concentration of bed load and suspended load together with the flow and channel characteristics.

2. Velocity distribution depends primarily upon the nature of the turbulence of the flow. The turbulence in turn depends upon the variables listed so that it becomes an additional dependent variable which must not be included.

3. Temperature influences only the viscosity of the fluid. Therefore it should not be included.

4. Aging of the channel results in sorting of the bed material and depositing of a colloidal coating which makes the channel smoother. This reflects the combined effect of σ , f' , and fc' which have already been listed.

The most general relationship of all the variables may be expressed as follows:

$$\phi(W, D, d, \sigma, d', \sigma', f', f_c', V, G, \rho, \rho_s, \Delta\gamma, \mu, s)=0$$

When D, V, and ρ are chosen as repeating variables the following function of dimensionless parameters is obtained:

$$\phi_1\left(\frac{W}{D}, \frac{d}{D}, \frac{\sigma}{D}, \frac{d'}{D}, \frac{\sigma'}{D}, f', f_c', \frac{G}{D\rho V^3}, \frac{\rho_s}{\rho}, \underline{R}, \underline{F}, s\right) = 0 \quad (3.1)$$

in which $\underline{R}$ is the Reynold number $VD\rho/\mu$, and $\underline{F}$ is the Froude number $V/\sqrt{\frac{\Delta\gamma}{\rho}D} \doteq V/\sqrt{gD}$.

From Eq. (3.1) it is evident that the problem is a complex one. Fortunately, however, the density of sediment varies over a narrow range. Furthermore, the form of the particles is mostly rounded without sharp angles, due to the attrition while rolling along with the fluid. Therefore, with these considerations Eq. (3.1) can be simplified to:

$$\phi_2\left(\frac{W}{D}, \frac{d}{D}, \frac{\sigma}{D}, \frac{d'}{D}, \frac{\sigma'}{D}, f_c', \frac{G}{D\rho V^3}, \underline{R}, \underline{F}, s\right) = 0 \quad (3.2)$$

Still further simplification may be accomplished by assuming that there is a definite relation between the characteristics of bed material and the sediment transported by the fluid, i.e. either $\frac{d}{D}$ and $\frac{\sigma}{D}$ or $\frac{d'}{D}$ and $\frac{\sigma'}{D}$ are dependent variables in Eq. (3.2). Hence Eq. (3.2) may be written as:

$$\phi_3\left(\frac{W}{D}, \frac{d}{D}, \frac{\sigma}{D}, f_c', \frac{G}{D\rho V^3}, \underline{R}, \underline{F}, s\right) = 0 \quad (3.3)$$

Although Eq. (3.3) is still a hopelessly complex function for experimental investigation, it is possible to restrict the problem to particular cases as discussed in Chapter VI.

Chapter IV

THEORETICAL SHAPE OF STABLE CHANNEL CROSS-SECTIONS

The investigation of the shape of the stable channel cross-section was started with Lane's idea, as he stated:

> The slope of the bank must be sufficiently flat so that the component along it, of the force of gravity, when combined with the force of the water, is insufficient to dislodge the particles (20:135).

Two cases must be considered, (a) for the non-cohesive material and (b) for the cohesive material:

Shape for non-cohesive material

Consider a single particle A of Fig. 1. When the forces acting on this particle are in equilibrium, the resultant of the tractive force by water and the downward sliding force of gravity should be balanced by the resisting force of the particle. i.e.:

$$t^2 + (w \sin \alpha)^2 = r^2 \qquad (4.1)$$

in which w is the weight of the particle in water, r is the resisting force of the particle against movement, and t is the tractive force of flowing water on the particle.

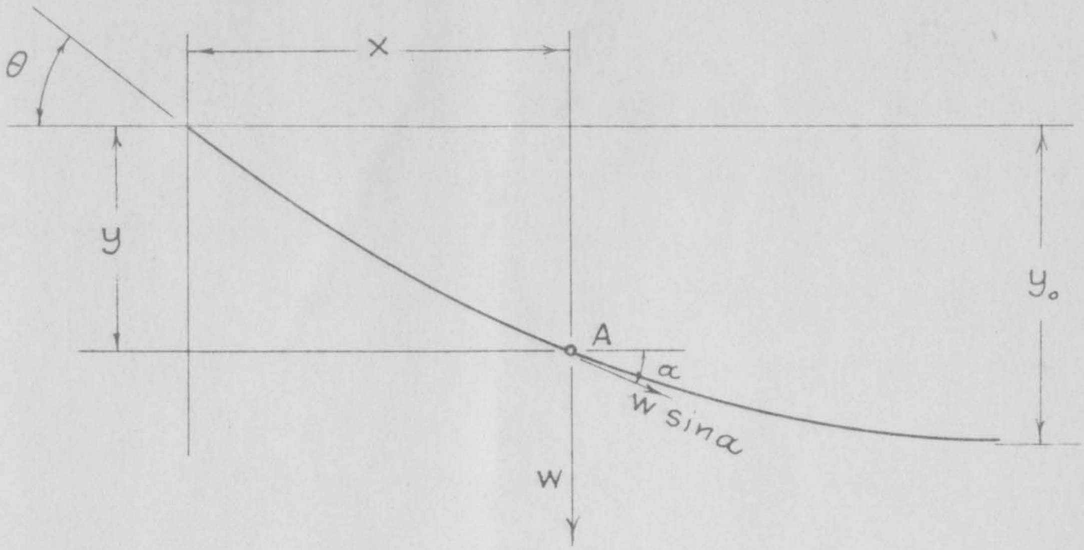


Fig. 1.—Channel cross-section

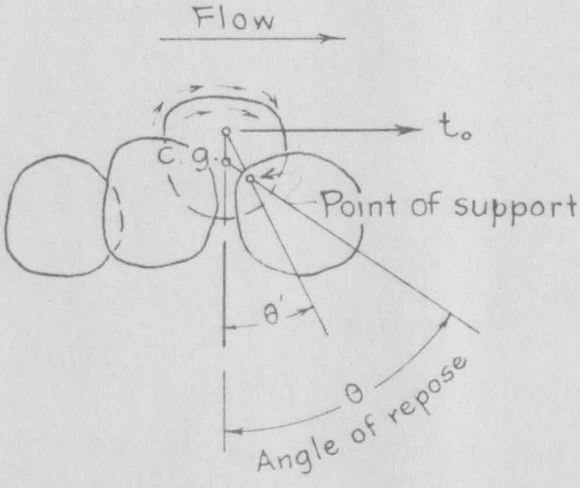


Fig. 2.—Equilibrium of an individual particle on a horizontal plane when the surface drag is of major importance.

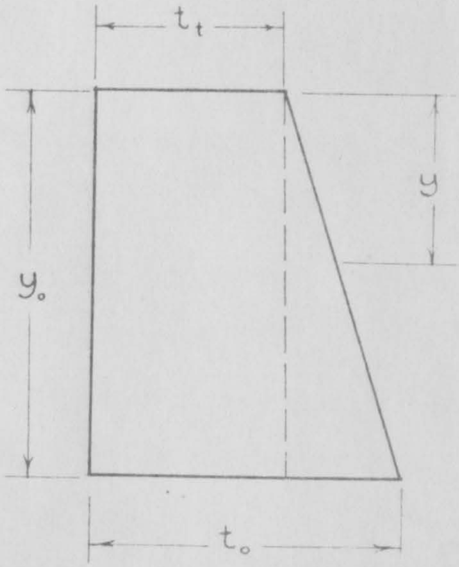


Fig. 3.—Variation of resisting forces of particles with depth.

Assume

$$t = t_o \frac{y}{y_o} \tag{4.2}$$

in which t_o is the tractive force on a single particle at the lowest point of the channel; and y_o is the maximum depth of water.

Further assume that when $y = o$, $\alpha = \theta$ the angle of repose of saturated particles, and $t = o$.

From Eq. (4.1):

$$(w \sin \theta)^2 = r_t^2$$

i.e.
$$r_t = w \sin \theta \tag{4.3}$$

in which r_t is the tractive force on a particle in the bank at water surface level.

When $y = y_o$:

$$\alpha = o \text{ and } t = t_o$$

From Eq. (4.1) $t_o^2 = r_o^2$. (4.4)

From the theory of the boundary layer, it is well known that there is a laminar sublayer existing above the surface of an hydrodynamically smooth boundary, even though the flow is turbulent. Because most alluvial irrigation canals may be considered smooth, it may be assumed that all the particles of the bed and bank are within this laminar sublayer. Therefore, surface drag and deformation drag dominate rather than form drag as the water flows over the particles. A schematic diagram of the forces involved is shown in Fig. 2. When these forces are in equilibrium,

$$t_o = w \tan \theta' = K w \tan \theta$$

According to the data from the experiments by White (27), K is between 0.3 and 0.4. Therefore, assuming $K = 1/3$,

$$t_o = \frac{1}{3} w \tan \theta$$

From Eq. (4.4) $t_o^2 = r_o^2 = (\frac{1}{3} w \tan \theta)^2$

or

$$r_o = \frac{1}{3} w \tan \theta \qquad (4.5)$$

If it is further assumed that r varies linearly with the depth, see Fig. 3, at point A:

$$r = r_t + (r_o - r_t) \frac{y}{y_o}$$

$$= w \sin \theta + (1/3\ w \tan \theta - w \sin \theta) \frac{y}{y_o}$$

$$= w \sin \theta \left[1 + \left(\frac{1}{3 \cos \theta} - 1 \right) \frac{y}{y_o} \right] \qquad (4.6)$$

Eq. (4.6) now may be substituted in Eq. (4.1) and divided by t_o^2, so that every term becomes dimensionless to yield

$$\frac{t^2}{t_o^2} + \frac{(w \sin \alpha)^2}{t_o^2} = \frac{w^2 \sin^2 \theta}{t_o^2} \left[1 + \left(\frac{1}{3 \cos \theta} - 1 \right) \frac{y}{y_o} \right]^2$$

From Eq. (4.2) and Eq. (4.5):

$$\left(\frac{y}{y_o} \right)^2 + 9 \left(\frac{\sin \alpha}{\tan \theta} \right)^2 = 9 \cos^2 \theta \left[1 + \left(\frac{1}{3 \cos \theta} - 1 \right) \frac{y}{y_o} \right]^2$$

therefore

$$\sin \alpha = \sin \theta \sqrt{ \left[1 + \left(\frac{2}{3 \cos \theta} - 1 \right) \frac{y}{y_o} \right] \left[1 - \frac{y}{y_o} \right] }$$

and

$$\frac{dy}{dx} = \tan\alpha = \frac{d\,(y/y_o)}{d\,(x/y_o)}$$

or

$$\frac{d\,(x/y_o)}{d\,(y/y_o)} = \frac{1}{\tan\alpha} = \frac{\sqrt{1 - \sin^2\alpha}}{\sin\alpha} \tag{4.7}$$

then

$$\frac{d\left(\frac{x}{y_o}\right)}{d\left(\frac{y}{y_o}\right)} = \frac{\sqrt{1 - \sin^2\theta\left[1 + \left(\frac{2}{3\cos\theta} - 1\right)\frac{y}{y_o}\right]\left[1 - \frac{y}{y_o}\right]}}{\sin\theta\sqrt{\left[1 + \left(\frac{2}{3\cos\theta} - 1\right)\frac{y}{y_o}\right]\left[1 - \frac{y}{y_o}\right]}}$$

finally

$$\frac{x}{y_o} = \int_o^{y/y_o}\sqrt{\frac{\csc^2\theta}{\left[1 + \left(\frac{2}{3}\cos\theta - 1\right)\frac{y}{y_o}\right]\left[1 - \frac{y}{y_o}\right]} - 1}\; d\left(\frac{y}{y_o}\right) \tag{4.8}$$

By graphical integration, the results of Eq. (4.8) are presented in Table 1 and in Fig. 4.

Shape for cohesive material

The following study was initiated in 1947 by Fai (7). The main assumption which he made to simplify the mathematical work was to consider r, the resisting force of the particles against scouring, a constant along the circumference of the cross-section. The following development shows that this assumption is evidently not true for non-cohesive material:

Combining Eqs. (4.3) and (4.5) and letting $t_t = t_o$,

$$w \sin\theta = 1/3\, w \tan\theta$$

which means

$$\cos\theta = 1/3$$

or θ must equal $70\frac{1}{2}^{\circ}$. Therefore, the foregoing assumption holds only for $\theta = 70\frac{1}{2}^{\circ}$.

For cohesive material, however,

$$r = f_c + f_f$$

where f_c is the cohesive force between particles and f_f is the frictional force between particles. Due to the meager knowledge of the cohesion of soil particles r becomes an unknown quantity. Therefore as an approximation it may be assumed that r is constant along the perimeter of the channel.

From Eq. (4.1) it may be seen that:

when $\quad y = o,\ \alpha = \theta,\ t = o,$

and $\quad \sin\theta = \frac{r}{W} \qquad (4.9)$

When $\quad y = y_o,\ \alpha = o,\ t = o,$

and $\quad t_o^2 = r^2 \qquad (4.10)$

Dividing Eq. (4.1) by Eq. (4.10),

$$\left(\frac{t}{t_o}\right)^2 = \frac{r^2 - (w \sin\alpha)^2}{r^2}$$

or

$$\left(\frac{t}{t_o}\right)^2 = \frac{(r/w)^2 - (\sin^2\alpha)}{(r/w)^2}$$

Substituting Eqs. (4.2) and (4.9) into the above equation:

$$\left(\frac{y}{y_o}\right)^2 = \frac{(\sin\theta)^2 - (\sin\alpha)^2}{(\sin\theta)^2}$$

Hence

$$\sin \alpha = \sin \theta \sqrt{1 - (y/y_o)^2} \qquad (4.11)$$

Finally, substituting Eq. (4.11) into Eq. (4.7):

$$\frac{d\left(\frac{x}{y_o}\right)}{d\left(\frac{y}{y_o}\right)} = \frac{\sqrt{1 - \sin^2\theta \;\left[1 - (y/y_o)^2\right]}}{\sin\theta \;\sqrt{1 - (y/y_o)^2}}$$

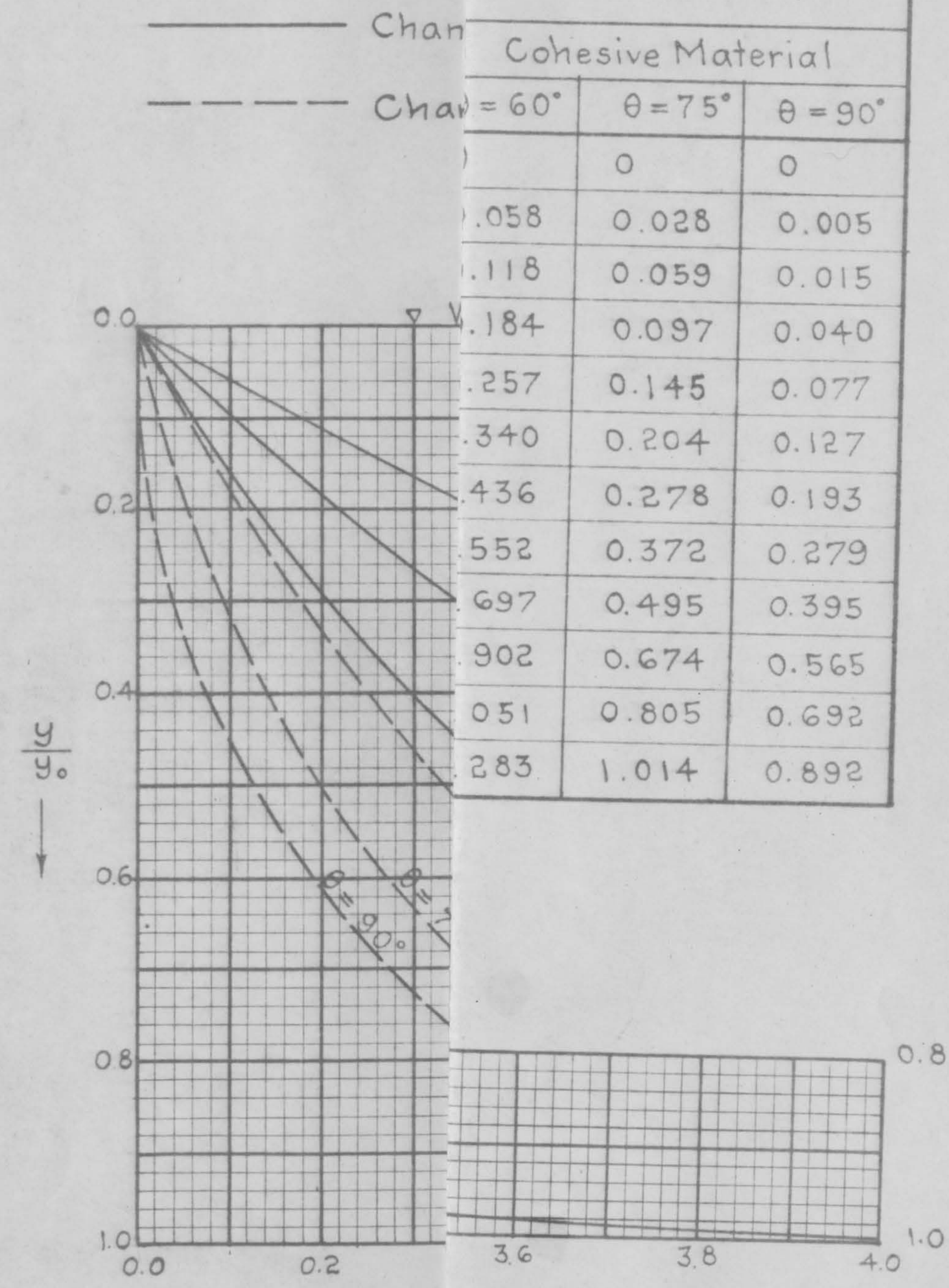
or

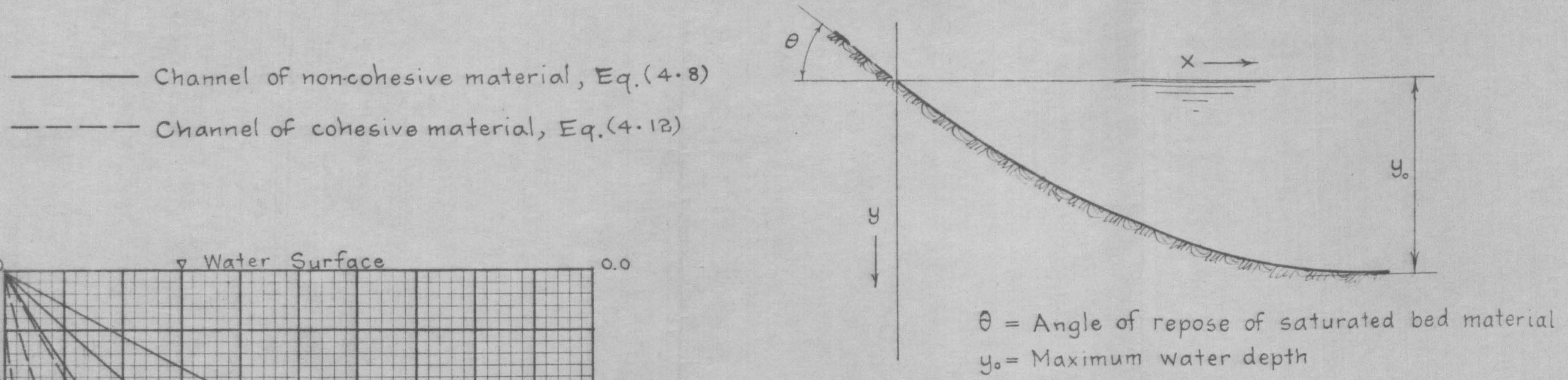
$$\frac{x}{y_o} = \int_o^{\frac{y}{y_o}} \sqrt{\frac{\csc^2\theta}{1 - (y/y_o)^2} - 1} \;\; d\,(y/y_o) \qquad (4.12)$$

This equation may be integrated graphically. The results of which are presented in Table 1 and Fig. 4.

tes of theoretical
channels

Cohesive Material		
= 60°	θ = 75°	θ = 90°
)	0	0
).058	0.028	0.005
).118	0.059	0.015
.184	0.097	0.040
.257	0.145	0.077
.340	0.204	0.127
.436	0.278	0.193
.552	0.372	0.279
.697	0.495	0.395
.902	0.674	0.565
051	0.805	0.692
.283	1.014	0.892





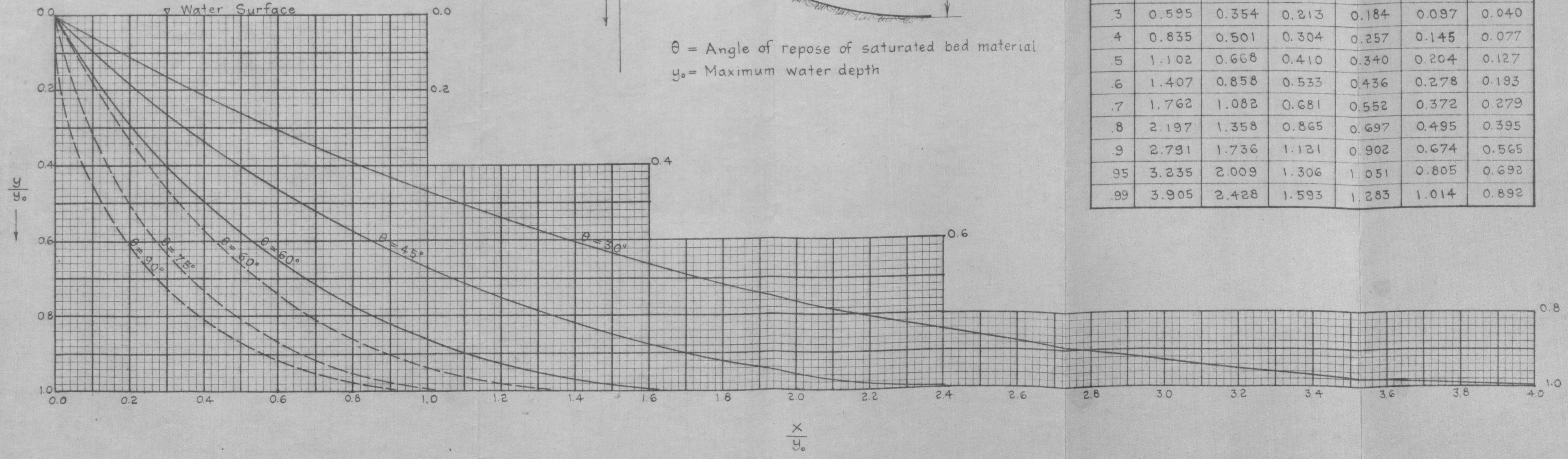


Fig. 4.—Theoretical cross-sections of stable channels

Table I.—Dimensionless coordinates of theoretical cross-sections of stable channels

$\frac{y}{y_o}$	x/y_o					
	Non-Cohesive Material			Cohesive Material		
	θ = 30°	θ = 45°	θ = 60°	θ = 60°	θ = 75°	θ = 90°
0	0	0	0	0	0	0
.1	0.181	0.106	0.062	0.058	0.028	0.005
.2	0.378	0.223	0.133	0.118	0.059	0.015
.3	0.595	0.354	0.213	0.184	0.097	0.040
.4	0.835	0.501	0.304	0.257	0.145	0.077
.5	1.102	0.668	0.410	0.340	0.204	0.127
.6	1.407	0.858	0.533	0.436	0.278	0.193
.7	1.762	1.082	0.681	0.552	0.372	0.279
.8	2.197	1.358	0.865	0.697	0.495	0.395
.9	2.791	1.736	1.121	0.902	0.674	0.565
.95	3.235	2.009	1.306	1.051	0.805	0.692
.99	3.905	2.428	1.593	1.283	1.014	0.892

Chapter V

DATA ANALYSIS

As indicated in the preceding two chapters, experimental data are needed to establish the general function given in Eq. (3.3) and to determine the validity of the theoretical equations which have been derived.

Dimensionless Functions

The most logical method of utilizing experimental data would be to analyse them according to the dimensionless parameters in Eq. (3.3). Unfortunately, however, the data on existing canals are incomplete so that it is not possible to compute all of the parameters. The information most generally missing is the temperature of the water and the size and concentration of the sediment. Therefore, it has been necessary in this study to use only the data for water that is essentially clear and to estimate the temperature based upon weather records.

Two kinds of bed material may give water that is nearly clear, namely, those with particles coarse enough that the flowing water cannot disturb them and those with compact graded sediment coated with settled colloid of high cementing value. None of the data were used for coarse material in existing canals, because sufficient information was not available on the description of the bed material. Furthermore, the coarse material is usually accompanied by fine material due to decomposition and disintegration.

In view of the foregoing, the data of canals used in this analysis are from canals of cemented fine material with almost clear water. The general dimensional analysis gives Eq. (3.3):

$$\phi_3\left(\frac{W}{D}, \frac{d}{D}, \frac{\sigma}{D}, f_c', \frac{G}{D\rho V^3}, \underline{R}, \underline{F}, s\right) = 0$$

Under the above mentioned condition, $\frac{G}{D\rho V^3} = 0$ and f_c' is always a high value and may be neglected. Then, Eq. (3.3) can be reduced to:

$$\phi_4\left(\frac{W}{D}, \underline{R}, \underline{F}, s, \frac{d}{D}, \frac{\sigma}{D}\right) = 0 \tag{5.1}$$

For highly cohesive material, the nearly-clear water can hardly change the width of the channel and the average water depth. Hence $\frac{W}{D}$ is largely artificial rather than controlled by flowing water and it probably does not represent the shape of the channel. This statement is consistent with the results of the theoretical channel shape derived in Chapter IV. That is, the curves of Fig. 4 for cohesive material show that a flat portion of uncertain length is always present in the central part of the channel. Therefore, in this case

$\frac{W}{D}$ is considered to be insignificant. Eq. (5.1) then becomes:

$$\phi_5\left(\underline{R}, \underline{F}, s, \frac{d}{D}, \frac{\sigma}{D}\right) = 0 \qquad (5.2)$$

Eq. (5.2) represents the functional relationship in a form as complete as can be obtained by dimensional analysis and logical reasoning. It is therefore necessary to investigate available theoretical relationships which will further develop the function.

Because open channel flow is primarily a gravity phenomenon, and because the velocity varies largely as the square root of the channel slope, $\underline{F}$ and s may be combined into one parameter as $\underline{F}/\sqrt{s}$. Further proof of the validity of such a step is found in the Chezy equation arranged for wide channels:

$$V = C\sqrt{Ds}$$

or

$$\frac{V}{\sqrt{Ds}} = C$$

So that

$$\frac{F}{\sqrt{s}} = \frac{C}{\sqrt{g}} \qquad (5.3)$$

Hence, Eq. (5.2) may be written as:

$$\phi_6\left(\underline{R}, \frac{F}{\sqrt{s}}, \frac{d}{D}, \frac{\sigma}{D}\right) = 0 \qquad (5.4)$$

or

$$\phi_6\left(\underline{R}, \frac{c}{\sqrt{g}}, \frac{d}{D}, \frac{\sigma}{D}\right) = 0 \qquad (5.5)$$

Evidently, this function expresses the variation of the resistance coefficient with the Reynolds number and the bed material.

Because flow in open channels is a boundary layer phenomenon, it is to be expected that the results of previous investigators should be applicable to this problem. It may be recalled that the Kármán-Schoenherr resistance equation for flow past a smooth boundary is

$$\frac{1}{\sqrt{C_f}} = 4.13 \log (\underline{R}\ C_f) \tag{5.6}$$

where C_f is the resistance coefficient. Likewise a similar, and perhaps more common, equation for turbulent flow in smooth pipes is the equation of Kármán-Prandtl:

$$\frac{1}{\sqrt{\underline{f}}} = 2 \log (R \sqrt{\underline{f}}) - 0.8 \tag{5.7}$$

where $\underline{f}$ is the Darcy-Weisbach resistance coefficient in the equation:

$$\frac{h_f}{L} = \underline{f} \frac{1}{4R} \frac{V^2}{2g}$$

in which R is the hydraulic radius of the flow in pipe. For flow in open channels this equation becomes equal to the slope s and may be rearranged as

$$\frac{V}{\sqrt{gRs}} = \frac{\sqrt{8}}{\sqrt{\underline{f}}}$$

which, if the average depth is set equal to the hydraulic radius, is equal to Eq. (5.3) so that

$$\frac{\sqrt{8}}{\sqrt{\underline{f}}} = \frac{C}{\sqrt{g}} \tag{5.8}$$

From Eq. (5.8), Eq. (5.7) can be written as:

$$\frac{C}{\sqrt{g}} = 5.66 \log (\underline{R}\ \sqrt{g}/C) + 0.292 \tag{5.7a}$$

Because of the similarity between flow in closed conduits and flow in open conduits, it is probable that Eq. (5.5) can best be rewritten as:

$$\phi_7\left(\frac{R\sqrt{g}}{C}, \frac{C}{g}, \frac{d}{D}, \frac{\sigma}{D}\right) = 0 \tag{5.9}$$

which will evidentally take a logarithmic form such as

$$\frac{C}{\sqrt{g}} = K_1 \log \frac{R\sqrt{g}}{C} + K_2 \tag{5.10}$$

K_1 and K_2 being constants which must be determined experimentally. The parameters describing the bed material must act as third and fourth variables.

To establish the validity of this general function and to determine the magnitude of the constants in Eq. (5.10), data were selected from the limited information available on stable channels in the United States--namely, data from the St. Clair River taken from the work of Li (22) in his master's thesis at the State University of Iowa and data on various irrigation canals taken from Scobey's reports (24). To determine the probable viscosity of the water, the mean water temperature was estimated from the mean air temperature during channel-flowing season, Table 5. All of the elementary data are arranged in Table 2. From Table 2, the dimensionless parameters were computed and arranged in Table 3.

The computed results are plotted in Fig. 5 with semi-logarithmic coordinates to conform to Eq. (5.10).

Table 2.--DATA ON STABLE CHANNELS

No.	Channel	Bank and bed material	Q c.f.s.	A ft.^{2}	V ft./sec.	W ft.	S ft./ft.	ν ft.^{2}/sec. in 10^{-5}
1.	Main channel above Ecarte, St. Clair River	Clay	189,370	65,500	2.90	1948	0.00002	1.46
2.	Main channel below Ecarte	Softer clay	178,000	68,500	2.60	2045	0.000019	1.46
3.	South channel above Bassatt	Fine sediment	68,000	33,500	2.03	1320	0.00002	1.46
4.	South channel below Bassatt	do	56,500	28,500	1.98	944	0.000015	1.46
5.	Middle channel, St. Clair River	do	33,800	18,400	1.84	692	0.000015	1.46
6.	Grand Canal, Arizona	Clay loam	161.8	59.6	2.72	29.0	0.000438	1.05
7.	Lateral 7, Turlock, Calif.	Compact sand	15.4	23.0	0.67	21.0	0.000098	1.21
8.	Main, Turlock, Calif.	do	35.0	35.2	0.99	28.0	0.000263	1.21
9.	Rist and Goss Ditch, Colo.	Heavy loam	3.3	3.6	0.91	6.3	0.00034	1.31
10.	Old Barnes Ditch, Colo.	Firm earth	9.9	8.5	1.17	9.0	0.00032	1.31
11.	Jarbeau Power Ditch, Colo.	Clay loam	32.3	16.4	1.96	13.6	0.00049	1.21
12.	Louden Ditch, Colo.	Compact sand	62.0	37.3	1.66	25.0	0.00038	1.31
13.	Masa Lateral, Colo.	Fine sediment	40.3	27.4	1.47	14.7	0.00026	1.21
14.	Boulder and White Rock, Colo.	Grade sediment	3.2	3.21	1.0	7.2	0.001246	1.21
15.	Billings Land & Irr. Company, Mont.	Clay loam	167.6	68.4	2.45	24.0	0.000230	1.21
16.	Do Lateral 2, Mont.	Sandy loam	6.37	8.2	0.78	8.4	0.000175	1.21
17.	Bitter Root Valley Irr. Company, Mont	Hard pan	95.3	59.8	1.58	27.0	0.00020	1.31

Table 2.--DATA ON STABLE CHANNELS -- Continued

No.	Channel	Bank and bed material	Q c.f.s.	A ft.^{2}	V ft./sec.	W ft.	S ft. / ft.	ν ft.^{2}/sec. in 10^{-5}
18.	Bitter Root Valley Irr. Co. Mont. (different reach)	Fine sediment	92.8	56.8	1.63	27.0	0.000262	1.31
19.	Do	Do	112.5	67.6	1.85	26.0	0.000215	1.31
20.	Do	Graded sediment	112.5	56.4	2.00	26.5	0.000312	1.31
21.	Billings Land & Irr. Co., Mont.	Benton shale	148.7	79.2	1.88	26.5	0.000152	1.21
22.	Do	Billings clay	169.5	66.2	2.56	25.0	0.00033	1.21
23.	Do	Benton shale	102.2	50.9	2.00	21.5	0.000335	1.21
24.	Do	Clean soil	106.9	46.5	2.30	20.0	0.000295	1.21
25.	Farmers' canal, Nebr.	High cohesive clay	310.8	139.8	2.22	51.0	0.00017	0.93
26.	Bear River Lateral, Utah	Silt and moss	4.63	3.89	1.19	5.5	0.00075	1.21
27.	Solveson & Co., Utah	Clean sand	4.0	4.0	1.01	7.5	0.00056	1.21
28.	Logan Lateral, Utah	Compact sediment	0.56	1.03	0.54	7.2	0.00135	1.21
29.	Point Lookout Canal, Utah	Clayey loam	87.3	58.9	1.48	35.8	0.00027	1.21
30.	Bear River Lateral 2, Utah	Do	7.9	5.74	1.38	7.8	0.000875	1.21

Table 3.--COMPUTED DIMENSIONLESS PARAMETERS FOR STABLE CHANNELS

No.	V	$D = \frac{A}{W}$	S	ν in 10^{-5}	$F = \frac{V}{\sqrt{gD}}$	$F/\sqrt{S}$	$R = VD/\nu$ in 10^5	$R\sqrt{S}/F$ in 10^4
1.	2.90	34.1	0.00002	1.46	0.087	19.4	68.0	35.0
2.	2.60	33.6	0.000019	1.46	0.079	18.1	60.0	33.1
3.	2.03	25.5	0.00002	1.46	0.071	15.8	35.0	22.1
4.	1.98	30.5	0.000015	1.46	0.063	16.2	41.0	25.3
5.	1.84	26.7	0.000015	1.46	0.063	16.2	34.0	21.0
6.	2.72	2.05	0.000438	1.05	0.336	16.1	5.43	3.37
7.	0.67	1.10	0.000098	1.21	0.113	11.4	0.61	0.535
8.	0.99	1.26	0.000263	1.21	0.156	9.6	1.03	1.07
9.	0.91	0.57	0.00034	1.31	0.214	11.6	0.4	0.345
10.	1.17	0.945	0.00032	1.31	0.213	11.9	0.85	0.714
11.	1.96	1.21	0.00049	1.21	0.314	14.2	1.96	1.38
12.	1.66	1.49	0.00038	1.31	0.240	12.3	2.17	1.77
13.	1.47	1.86	0.00026	1.21	0.191	11.9	2.26	1.90
14.	1.00	0.446	0.001246	1.21	0.270	7.65	0.37	0.484
15.	2.45	2.85	0.000230	1.21	0.256	16.8	5.79	3.45
16.	0.78	0.98	0.000175	1.21	0.140	10.6	0.63	0.594
17.	1.58	2.21	0.00020	1.31	0.189	13.4	2.68	2.00
18.	1.63	2.10	0.000262	1.31	0.199	12.3	2.61	2.12
19.	1.85	2.60	0.000215	1.31	0.202	13.7	3.67	2.68
20.	2.00	2.13	0.000312	1.31	0.242	13.7	3.25	2.37

Table 3.--COMPUTED DIMENSIONLESS PARAMETERS FOR STABLE CHANNELS -- Continued

No.	V	D	S	ν in 10^{-5}	F	$F/\sqrt{S}$	R in 10^{5}	$R\sqrt{S}/F$ in 10^{4}
21.	1.88	2.99	0.000152	1.21	0.192	15.6	4.64	2.98
22.	2.56	2.65	0.00033	1.21	0.278	15.3	5.60	3.66
23.	2.00	2.37	0.000335	1.21	0.229	12.5	3.92	3.14
24.	2.30	2.32	0.000295	1.21	0.266	15.5	4.40	2.84
25.	2.22	2.74	0.00017	0.93	0.237	18.2	6.55	3.60
26.	1.19	0.708	0.00075	1.21	0.250	9.1	0.69	0.758
27.	1.01	0.533	0.00056	1.21	0.244	10.3	0.45	0.437
28.	0.54	0.143	0.00135	1.21	0.252	6.9	0.063	0.00913
29.	1.48	1.65	0.00027	1.21	0.204	12.4	1.99	1.60
30.	1.38	0.738	0.000875	1.21	0.283	9.6	0.85	0.886

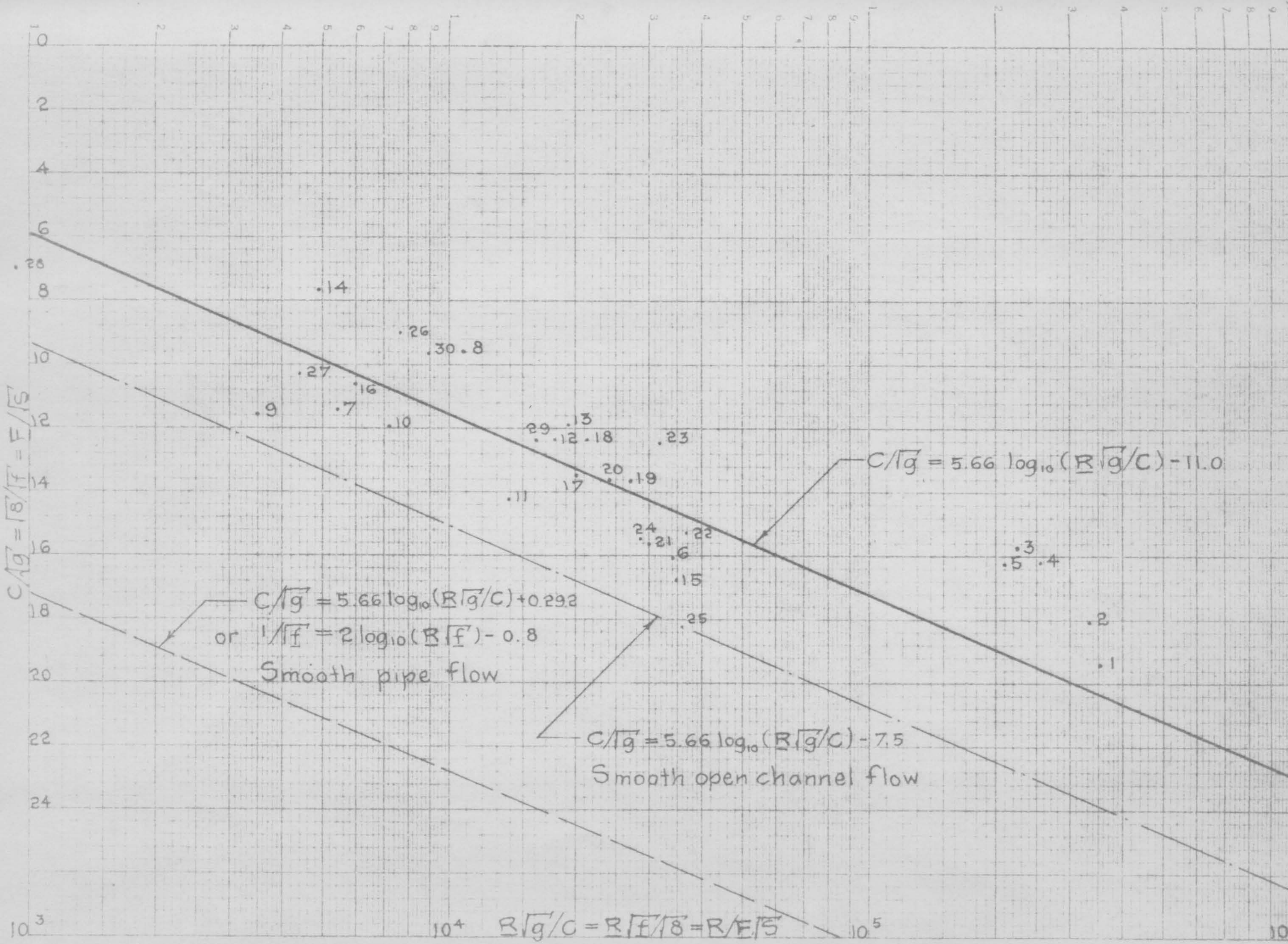


Fig 5. —Variation of $C/\sqrt{g}$ with $R\sqrt{g}/C$

The average curve drawn through the data follows the general form of Eq. (5.10) and establishes the constants so that

$$\frac{C}{\sqrt{g}} = 5.66 \text{ Log } \underline{R} \sqrt{g}/C - 11.0 \qquad (5.11)$$

Because data are not available on the bed material, it is not possible to draw curves of constant $\frac{d}{D}$ and the plotting can show only the variation of $\underline{R}$ with $\underline{F}/\sqrt{s}$.

To compare the data with the function for resistance in pipes, Eq. (5.7) is plotted as shown. It is to be expected that this equation should not agree exactly with the data because the equation is for pipes and the data are for open channels. Furthermore, the Eq. (5.7) is for smooth pipes whereas the data probably represent channels which are not smooth. Nevertheless, it is noteworthy that Eq. (5.7) for pipes and Eq. (5.11) representing the data are parallel. Evidently, the assumptions made in developing Eq. (5.10) are substantiated by field data.

Because the data probably represent channels that are not exactly smooth, it is to be expected that they all fall above a smooth-boundary curve which acts as a lower envelope. For this reason an estimated curve is drawn in such that all of the points fall above it.

$$\frac{C}{\sqrt{g}} = 5.66 \log \underline{R} \sqrt{g}/C - 7.5 \qquad (5.12)$$

This equation for smooth channels corresponds to Eq. (5.7) for smooth pipes.

Shape of the channel Cross-section

To check the theoretical shape of the cross-sections of stable channels, existing channel cross-sections were compared with the calculated cross-sections from Eq. (4.8) for non-cohesive material and Eq. (4.12) for cohesive material. The angle θ was estimated by the inclination of the upper portion of the existing channel cross-section. Although the determination of whether the bed material is cohesive or non-cohesive is questionable, an estimate was made on the basis of the meager data available.

The existing cross-sections which were chosen for comparison with the theoretical ones are for both cohesive and non-cohesive bed material and have various values of θ. For cross-sections number 1 to 4, which were measured in India (1:84), the bed material was chosen as non-cohesive and θ ranged from 30° to 45°. Cross-sections number 5 to 10 were measured in California by Mr. C. H. Rohwer, Senior Irrigation Engineer, Soil Conservation Service, U.S.D.A. Of these, numbers 5 and 6 were in material that was considered non-cohesive and θ was 45°.

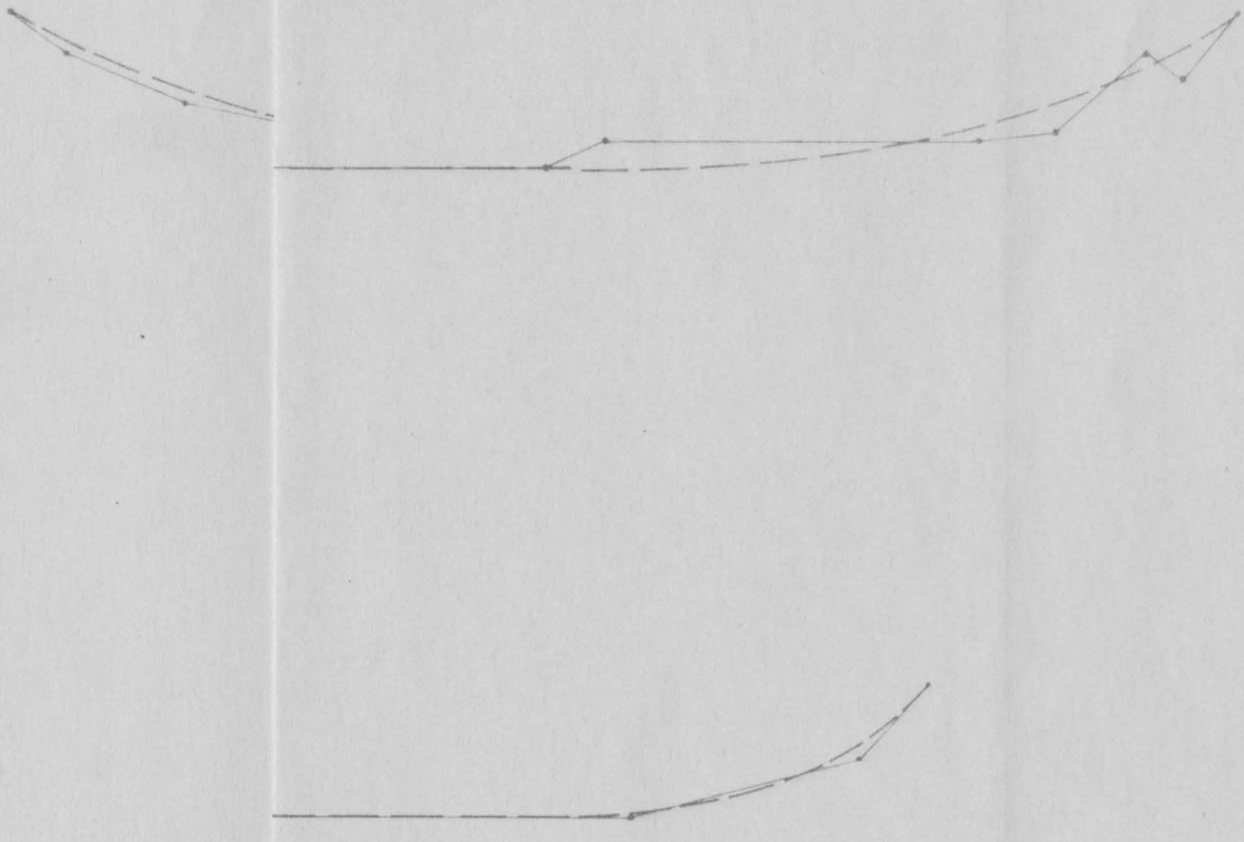
Cross-sections number 7 to 10, however, were estimated to be in cohesive material and θ was found to vary from 60° to 90°. Finally, cross-sections number 11 and 12 were measured by the writer in 1944 while working in Kansu, China. For these cases the bed material was assumed to be cohesive and θ was 60° and 75°.

Table 4 is a tabulation of the information on the foregoing canal cross-sections. The graphical comparison of existing and theoretical cross-sections are shown in Figs. 6 to 11. For lack of more exact information, straight lines were drawn to connect the data points.

The close general agreement between the theoretical and the actual cross-section shapes substantiates the use of Eqs. (4.8) and (4.12) for design purposes, at least as a first approximation.

Table 4.--CANAL CROSS-SECTIONS

No.	Canal	Location of the cross-section	Bed material	Cohesive or non-cohesive	θ
1.	Rohri Canal, Sind, India	64.6 miles from intake	Fine sand	Non-cohesive	30°
2.	Do	118.2 miles from intake	Do	Do	45°
3.	Do	134.3 miles from intake	Do	Do	30°
4.	Do	154.2 miles from intake	Do	Do	30°
5.	Main canal, Anderson Cottonweed, Calif.	Gage station	Gravelly loam	Do	45°
6.	Highline, Turlock, Calif.	2.6 miles upstream from East Avenue bridge	Sandy loam	Do	45°
7.	Highline, Turlock, Calif.	Head gate	Clay loam	Cohesive	90°
8.	West side main, Imperial, Calif.	Gage station	Sand and clay	Do	75°
9.	Briggs Ditch, Fresno, Calif.	Upper gage station	Fine sandy loam	Do	60°
10.	Briggs Ditch, Fresno, Calif.	Jensen Avenue station	Do	Do	75°
11.	Hulu Ditch, Kansu, China	1640 ft. from intake	Sand and clay	Do	60°
12.	Nanchuan Ditch, Kansu, China	Head gate	Clay	Do	75°



— Theoretical cross-section

2.

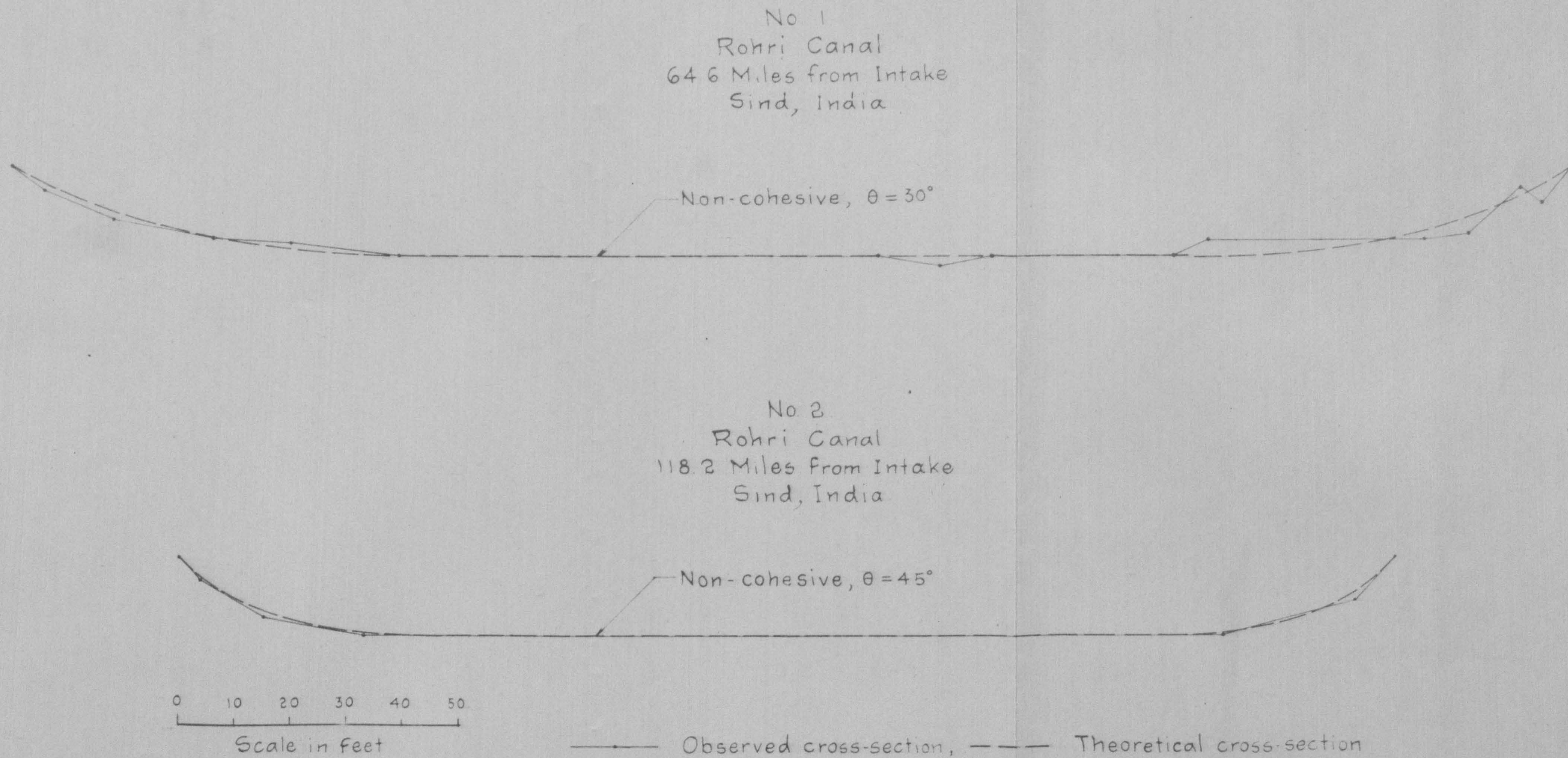


Fig. 6.—Canal cross-sections, Nos. 1 & 2.

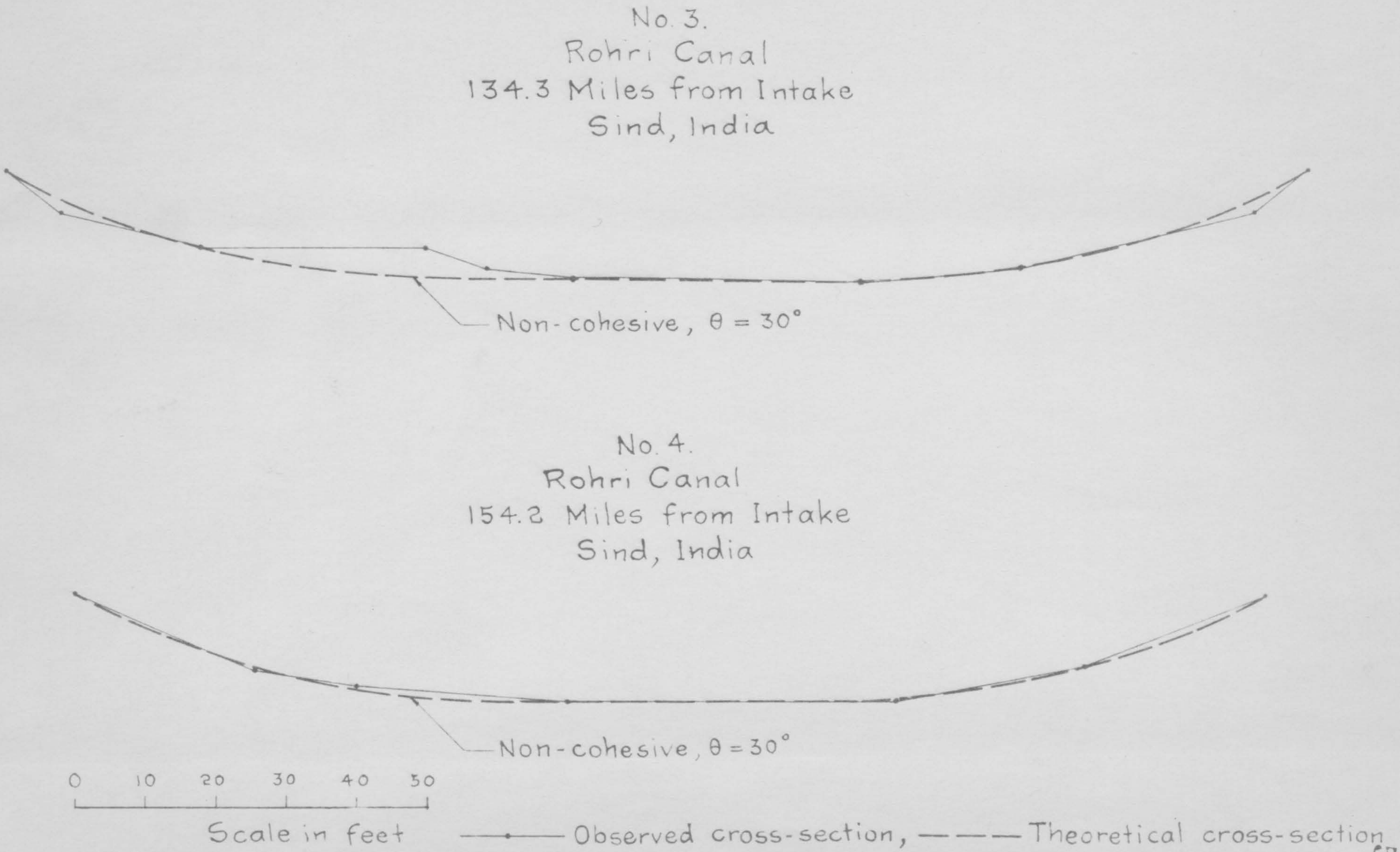


Fig. 7.—Canal cross-sections, Nos. 3 & 4

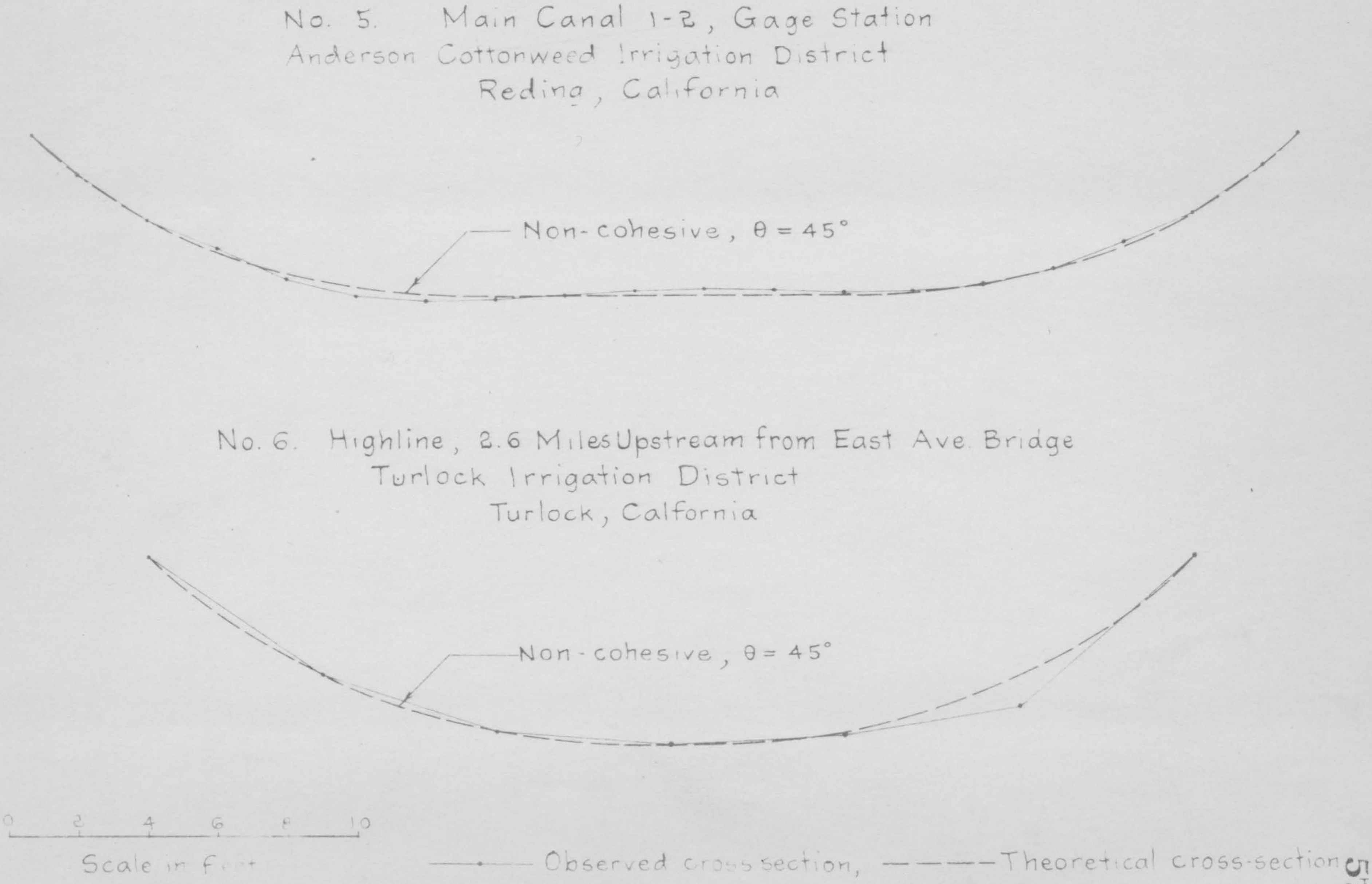


Fig 8.—Canal cross-sections, Nos 5 & 6.

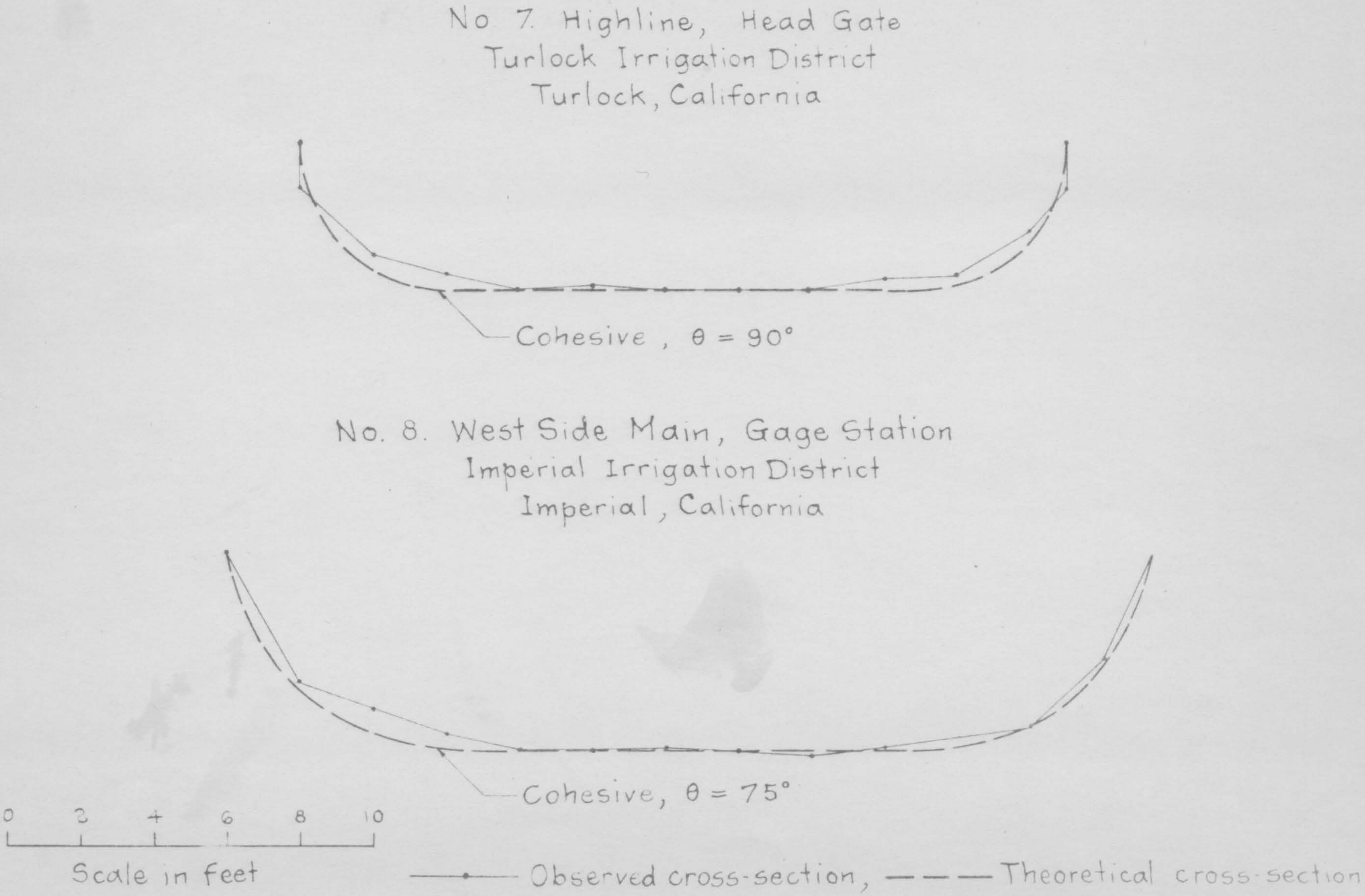


Fig 9.—Canal cross-sections, Nos. 7 & 8

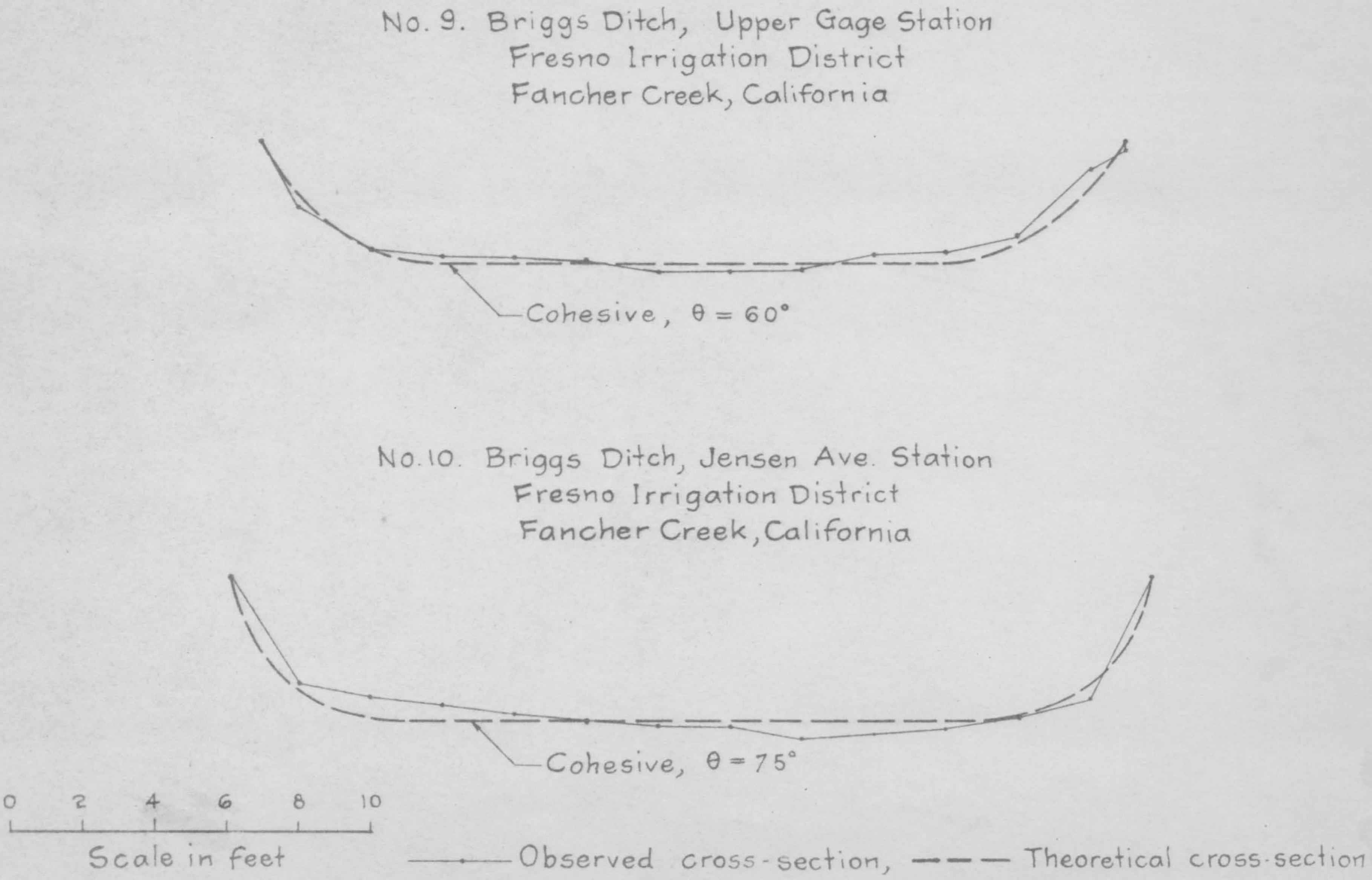


Fig. 10.—Canal cross-sections, Nos. 9 & 10.

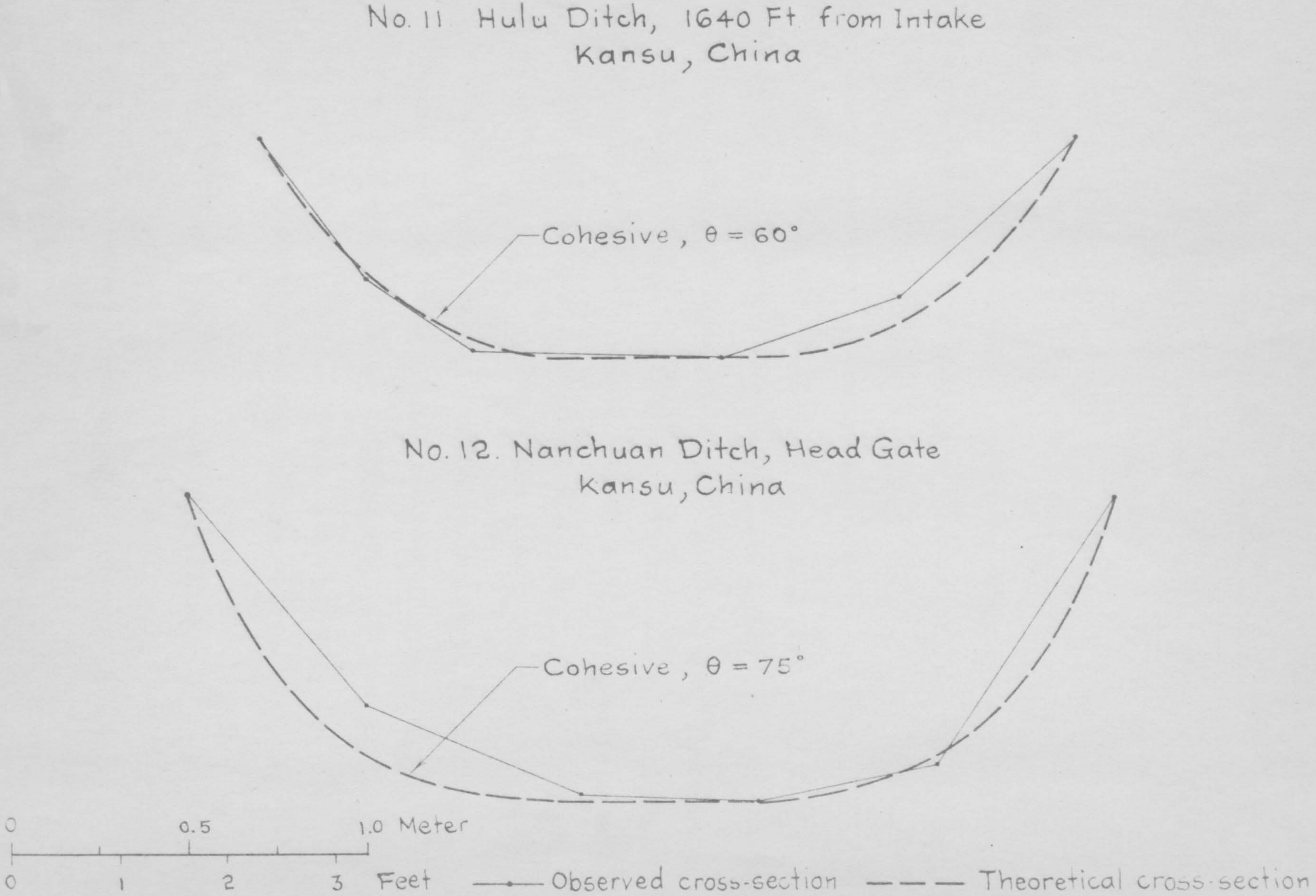


Fig. 11.—Canal cross-sections, Nos. 11 & 12.

Chapter VI

DISCUSSION

Of particular concern to engineers in this field is the problem of designing a channel that is safe at all times. In other words, a channel that will neither be scoured to an undesireable shape nor filled up by settling sediment. It should be remembered, however, that the only acceptable condition is not necessarily actual non-scouring and non-silting. Rather, if the degree of scouring and silting stays within the limits of keeping the channel in good shape, it can be said to be in a safe condition. Although the best condition is when equilibrium is reached, i.e. the rate of scour equals the rate of deposit, it is not possible to keep this equilibrium condition at all times because no channel can avoid fluctuations of discharge. Therefore, it is best to investigate the situation thoroughly and keep the design condition prevailing as much of the time as possible.

Suggestions for designing stable channels:

For determining the general characteristics of flow, Fig. 5 can be quite helpful, although due to limited data, the plot is not complete. By investigating the material in which the channel will be constructed and comparing it with the information listed in Table 2, the design engineer may get a general idea about how the condition under consideration varies with the average condition indicated by Eq. (11) in Fig. 5. This figure gives more appropriate information than any empirical equation derived from average data.

To determine the cross-sectional shape of a channel, Table 1 or Fig. 4 may be used. The only problem is to determine θ the angle of repose of the material. If it is possible, determining θ experimentally is most desirable. Ordinarily, however, the material is not homogeneous and the material in the bank may not be the same as that in the bed. In fact it may be difficult to determine whether the material belongs to the cohesive class or the non-cohesive class. In the end only average conditions should be used. Therefore the shape may best be approximated by a trapezoid based on the curves in Fig. 4. Such a trapezoidal shape will at least be more conducive to a stable condition than a shape designed by arbitrarily assuming the side slopes of the channel.

In view of this discussion, the following further items must also be kept in mind.

1. Time--It takestime to silt up a channel or to scour it into an undesireable shape. Likewise, once the equilibrium condition is disturbed, it takes time to bring it back to equilibrium. Therefore, non-equilibrium conditions which cause serious scouring or silting should be permitted only for a short period. Mild scouring or silting, on the other hand, can be permitted as long as no permanent damage is done.

2. Design discharge--Contrary to usual hydraulic design procedure, the maximum discharge is not the design discharge. Instead, the design discharge is the discharge which permits reasonable scouring and silting with equilibrium conditions prevailing during most of the water-flowing season. This design discharge may be called the dominant discharge.

3. Sediment characteristics--As pointed out in Chapter V, had more information on the bed material been available it would have been possible to draw a family of curves on Fig. 5 instead of one curve with a rather large range of scattered points. It is therefore evident that the characteristics of sediment would be very helpful in stable channel analysis. In most cases when Manning's equation or Chezy's equation is used for design work, it is felt that the choice of the resistance coefficient N_m or C is rather arbitrary, involving considerable guess work. Noway day's research on this problem and the

general problem of open channel flow is pointed in the direction of getting more precise information than is embodied in those former classic equations. It is logical, then, that this report should emphasize the importance of sediment characteristics. To this end cooperation among geologists, soil scientists, and hydraulic engineers is extremely important. For example, in treating suspended material, the electro-chemical effect cannot be neglected and in dealing with bed material, electron charge of very fine particles is the important factor of cohesiveness.

In studying the quantity of sediment transported by a given fluid, common practice has been to divide it into two classes for convenience of investigation, namely, the suspended load and the bed load. For this reason, various equations for these classes have been developed in modern engineering literature.

Kalinske (12, 13) used White's (27) experimental results and the basic physical principles of fluid dynamics to develop an equation for the rate of bed-load discharge. Einstein (6) concluded on the basis of observation that a given size of particle moves in a series of steps of definite length and frequency, and that the rate of transport depends upon the number of particles moving at that time. He assumed the probability that any one particle will begin to move in a given unit of time to be expressible in two ways: (a) in terms of the rate of transport, the size and relative weight of the particles, and a time factor equal to the ratio of the particle diameter to its velocity of fall; (b) in terms of the ratio of the forces exerted by the flow to the resistance of the particle to movement where the resistance is proportional to the immersed weight of the particle and the motivating force proportional to that given by the usual drag equation in terms of the fluid velocity at the elevation of the laminar sublayer. Equating these two forms of the probability relationship, Einstein developed a general function.

For suspended load, a fundamental equation has been developed by various investigators from the theory of fluid turbulence.

Theoretically speaking, any truly correct bed-load equation should be capable of extension to include the sediment transported in suspension. Likewise, any truly correct suspended-load equation should also include the bed load. Just above the bed, a continual interchange of material is occurring between the bed and the overlying fluid and in this locality it is obviously difficult to distinguish between the bed load and suspended load. If the function of total sediment transported could be expressed in explicit terms, the problem of designing stable channels would be much simpler.

<u>Recommendations for further research</u>

Although this report shows that considerable advance toward a solution of the phenomenon of stable channels has been made, this solution is still far from complete or adequate. As shown in the dimensional analysis of Chapter III and in the foregoing discussion, the problem involves many variables and complex relationships. In solving such a complex problem, the best approach is usually to attack the simplest case first. Hence, the non-cohesive material with comparatively coarse granular particles of nearly uniform size probably should be chosen for the first investigation. For this case Eq. (3.3) becomes:

$$\phi_9\left(\frac{W}{D}, \frac{d}{D}, \frac{G}{D\rho V^3}, \underline{R}, \underline{F}, s\right) = 0 \qquad (6.1)$$

Because it is very difficult to make a study of this nature in existing canals, it is more economical, faster, and more accurate to carry it on first as a laboratory investigation. For this purpose, a channel is needed in which may be placed the bed material at any desired slope. Because for a given discharge the sediment being transported is the amount required to reach equilibrium, it is necessary to be able to recirculate the sediment.

If the analysis in Chapter V is assumed to be correct, Eq. (6.1) may be written as:

$$\phi_{10}\left(\frac{W}{D}, \frac{d}{D}, \frac{G}{D\rho V^3}, \underline{R}, \underline{F}/\sqrt{s}\right) = 0 \tag{6.2}$$

Although the validity of such an assumption must be established by experiment it is possible that in this particular case, both $\frac{W}{D}$ and $\frac{G}{D\rho V^3}$ are dependent variables as discussed in the first part of this chapter. If such proves to be true then the following functions result:

$$\frac{W}{D} = \phi_{11}\left(\frac{d}{D}, \underline{R}, \underline{F}/\sqrt{s}\right) \tag{6.3}$$

and

$$\frac{G}{D\rho V^3} = \phi_{12}\left(\frac{d}{D}, \underline{R}, \underline{F}/\sqrt{s}\right) \tag{6.4}$$

Finally, although the exact determination of each of these functions depends upon laboratory research, the behavior in existing canals remains the final criterion by which conclusions founded on the relatively simple conditions of the laboratory must ultimately be tested.

Chapter VII

SUMMARY

In the past the problem of stable channels has been studied largely by empirical methods for immediate help in design work. Recently, however, there has been a tendency to attempt to base the design on the laws of sediment transportation and to express the equilibrium condition mathematically. Unfortunately, no perfect equation has yet been formulated. It is possible that an approach to this problem can be made by a combination of rational and experimental methods and that results can be expressed by dimensionless plots. As a summary of this report the following two statements may be made.

1. The general flow characteristics in stable channels may be expressed as functions of the Reynolds number, the Froude number, the water surface slope, and the characteristics of the sediment. Eq. (5.11) represents the average condition and may be used as a first approximation in design. Eq. (5.12) is assumed as a lower envelope for smooth channels. Both equations can be plotted as straight lines on semi-logarithmic paper and are parallel with the line representing the smooth

pipe flow as shown in Fig. 5.

2. The shape of the stable channel cross-section may be expressed mathematically. For practical application, the curves of Fig. 4 are available for immediate use and the tedious computations to obtain values from Eq. (4.8) or Eq. (4.12) are not required.

A P P E N D I X

NOTATION

In the following symbols the English system of units is used unless otherwise stated.

c -- sediment concentration in parts per 10,000 by weight

d -- mean sediment diameter in millimeters

f -- silt factor in Lacey-type equations

$\underline{f}$ -- Darcy - Weisbach resistance coefficient

f_c -- cohesive force between particles

f_f -- frictional force between particles

f' -- a sediment shape factor

f_c' -- a cohesion factor of sediment

h_f -- resistance loss in feet of water

g -- gravitational acceleration

k -- the ratio of V/v_a

m -- a coefficient in Kennedy-type equations

n -- an exponent in Kennedy-type equations

q -- rate of discharge per unit width of channel

r -- resisting force of a single particle against movement

r_t -- resisting force of a particle of the channel bank at the water surface

r_o -- resisting force of a particle at bottom of the channel

t -- tractive force of flowing water on a single particle

t_o -- tractive force on a single particle at the lowest point of the channel

v -- velocity of water in a certain region or at a point

v_a -- mean velocity along the vertical of mean depth

w -- weight of a single sediment particle within the water

x -- horizontal co-ordinate of cross-section

y -- water depth and vertical co-ordinate of cross-section

y_o -- maximum water depth

A -- area of channel cross-section

B -- width of channel bed

C -- Chezy resistance coefficient

C_f -- mean drag coefficient

D -- mean depth of water

$\underline{F}$ -- Froude number

G -- weight rate of sediment discharge

K -- a constant

L -- length of pipe

N -- Kutter coefficient of roughness

N_m -- Manning coefficient of roughness

N_a -- absolute coefficient of roughness introduced by Lacey

N_1 -- a coefficient of roughness introduced by Chatley

P -- wetted perimeter

Q -- rate of discharge of water

R -- hydraulic radius = A/P

$\underline{R}$ -- Reynolds number

$\underline{R}_W$ -- Reynolds number with surface width as the length variable

S -- 1,000 s

S_m -- 1,000,000 s

T -- tractive force per unit area

T_c -- critical tractive force

V -- mean velocity in a section

V_c -- critical velocity from stand point of non-silting and non-scouring

W -- width of water surface

W_a -- mean width

X -- a sediment parameter in Du Boy's equation

Z -- a sediment coefficient introduced by Griffith

α -- angle between horizontal line and a tangent to the curve

γ -- specific weight of water

$\Delta\gamma$ -- interface difference of specific weight

θ -- angle of repose of saturated material

μ -- dynamic viscosity

μ' -- coefficient of rubbing friction

ν -- kinematic viscosity

ρ -- density of fluid

ρ_s -- density of sediment

σ -- standard deviation of sizes of sediment

ϕ -- a function

Table 5.--MEAN TEMPERATURE DURING IRRIGATION SEASON

[The data of this table are from "Summaries of climatological data by sections, U. S. D. A. Weather Bureau Bulletin W, 1926."]

Month / Station	May	June	July	Aug.	Sept.	Average
Boulder, Colorado	56.4	66.0	70.8	70.6	63.2	65.4
Ft. Collins, Colorado	53.8	63.1	68.0	67.5	59.2	62.3
Ft. Morgan, Colorado	56.4	66.6	73.1	71.0	62.0	65.8
Rifle, Colorado	55.4	65.1	70.7	69.2	61.1	64.3
Logan, Utah	54.4	63.4	71.7	70.7	61.2	64.3
Billings, Montana	54.8	63.0	70.8	68.9	58.7	59.1
Como, Montana	51.6	59.6	65.8	63.9	54.5	84.4
Scottsbluff, Neb.	----	--	81.1	87.6	----	----
Sacramento, Calif.	Perennial flow			Annual ave.		60.0
Denair, Calif.	Do					59.8
Phoenix, Ariz.	Do					69.5

B I B L I O G R A P H Y

BIBLIOGRAPHY

1. Blench, T. and King, C. A preliminary note on the effect of dynamic shape on Lacey relations. India. Central Board of Irrigation. Annual report (Tech.) 27:75-109, 1941.

2. Bottomley, W. T. Discussions on alluvium rivers and canals and water supply of Kano. Institution of Civil Engineers, London. Minutes of proceedings, 237:512-15, 1933-34.

3. Buckley, A. B. The influence of silt on the velocity of water flowing in open channels. Institution of Civil Engineers, London. Minutes of proceedings, 216:183-298, 1922-23.

4. Chatley, H. River Control problems, (IV) Stable sectional form of alluvial rivers. Engineering, 146:740-2, Dec. 23, 1938.

5. __________ River flow problems, (IV) Bed traction and depth of streams. Engineering, 146:166, August 5, 1938.

6. Einstein, H. A. Formulas for the transportation of bed load. American Society of Civil Engineers. Transactions, 107:561-577, 1942.

7. Fai, C. Y. A study of the shape of the stable channel cross-section. (In Chinese) Chinese Society of Hydraulic Engineers. Hydraulic Engineering, 15:1:71-79.

8. Griffith, W. M. Further notes on silt transportation. Institution of Civil Engineers. Journal, November 1946:67-81.

9. __________ Silt transportation and its relation to regime channel sections. Institution of Civil Engineers. Journal, April 1944:107-120.

10. __________ A theory of silt and scour. Institution of Civil Engineers. Minutes of proceedings, 223:243-314, 1926-27.

11. __________ A theory of silt transportation. American Society of Civil Engineers. Transactions, 104:1734-86, 1939.

12. Kalinske, A. A. Criteria for determining sand transport by surface creep and saltation. American Geophysical Union. Transactions, 23:639-43, 1942.

13. __________ Movement of sediment as bed load in rivers. American Geophysical Union. Transactions, 28:615-20, 1947.

14. Kennedy, R. G. The prevention of silting in irrigation canals. Institution of Civil Engineers. Minutes of proceedings, 119:281-90, 1895.

15. King. C. Practical design formulas for stable irrigation channels. India. Central Board of Irrigation. Annual Report (Tech.) 31:52-61, 1943.

16. Lacey, G. A general theory of flow in alluvium. Institution of Civil Engineers. Journal, Nov. 1946:16-47.

17. __________ Regime flow in coherent alluvium. India Central Board of Irrigation. Publication No. 20:1-50, 1939.

18. __________ Stable channels in alluvium. Institution of Civil Engineers. Minutes of proceedings, 229:259-384, 1929-30.

19. __________ Uniform flow in alluvial river and canals. Institution of Civil Engineers. Minutes of proceedings, 237:421-53, 1933-34.

20. Lane, E. W. Stable channels in erodible material. American Society of Civil Engineers. Transactions, 102:123-194, 1937.

21. __________ and Kalinske, A. A. The relation of suspended to bed materials in rivers. American Geophysical Union. Transactions, 20:637-41, 1939.

22. Li, C. T. Comparison of Lacey's stable channel relations with the conditions in the St. Clair

and Lower Mississippi River. Master's Thesis, 1939, University of Iowa.

23. Lindley, E. S. Regime channels. India. Punjab Engineering Congress. Proceedings, 7:63-74, 1919.

24. Scobey, F. C. The flow of water in irrigation channels. Washington, Govt. Print. Off., 1915. 68 p. (U. S. Dept of Agriculture. Bulletin no. 194).

25. Straub, L. G. Approaches to the study of the mechanics of bed movement. Iowa University. Studies in engineering. Bulletin 20:178-92, 1940.

26. __________ Discharge and sediment relationships in an open channel. American Society of Civil Engineers. Civil Engineering, Nov. 1946: 490-91.

27. White, C. M. Equilibrium of grains on bed of stream. Royal Society, London. Proceedings, 174A:322-334, 1940.

28. Woods, F. W. A new hydraulic formula (for silting velocity--Kennedy's Data). Engineer, 143:646-648, June 17, 1927.