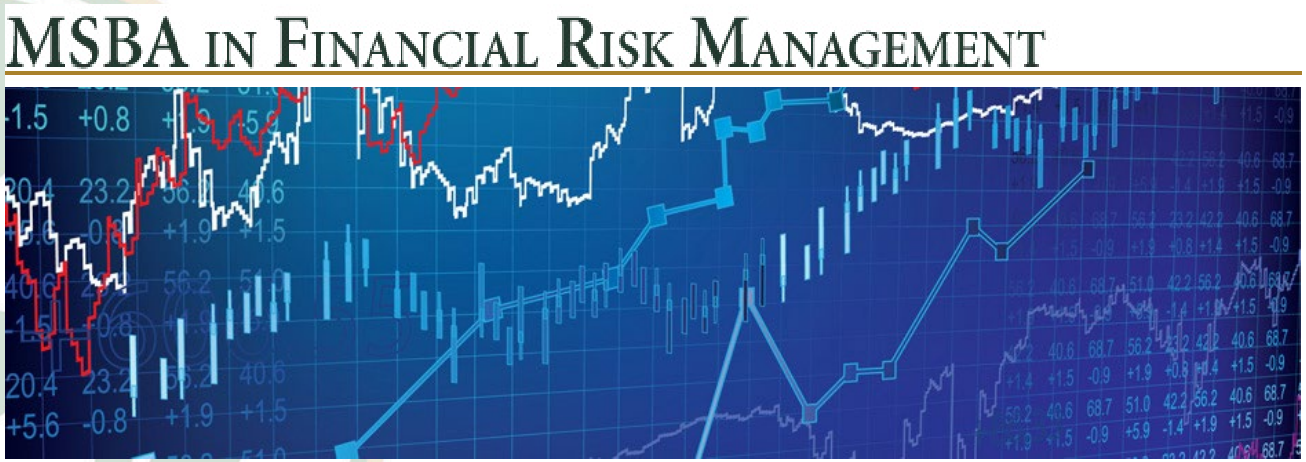
Financial Risk Management



Beyond Value At Risk

Coherent Risk Measure

A set of "risk-measure axioms"

"Well-behaved"

- Monotonicity: $V(Y) \leq V(X) \rightarrow \rho(X) \leq \rho(Y)$
- Translation invariance: $\rho(X+n) = \rho(X) - n$
- (Positive) homogeneity: $\rho(hX) = h\rho(X)$, $h>0$
- Subadditivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$

– Interpret the risk measure ρ: minimum cash that has to be added to a *risky* position to make this risky position acceptable

- VaR not sub-additive
  - Temptation to split up accounts or firms

Problem of VAR

- VaR is non-subadditive in general
  - E.g., two identical bonds A and B, each with a default probability of 4% and a loss of 100 if defaults
    - 95% VaR for A? for B?
    - Assuming independence, what is 95% VaR of the portfolio (A+B)?
    - How does the portfolio VaR compare to the sum of each bond's VaR?
  - VaR is sub-additive only in special situations (e.g., Normal distribution)

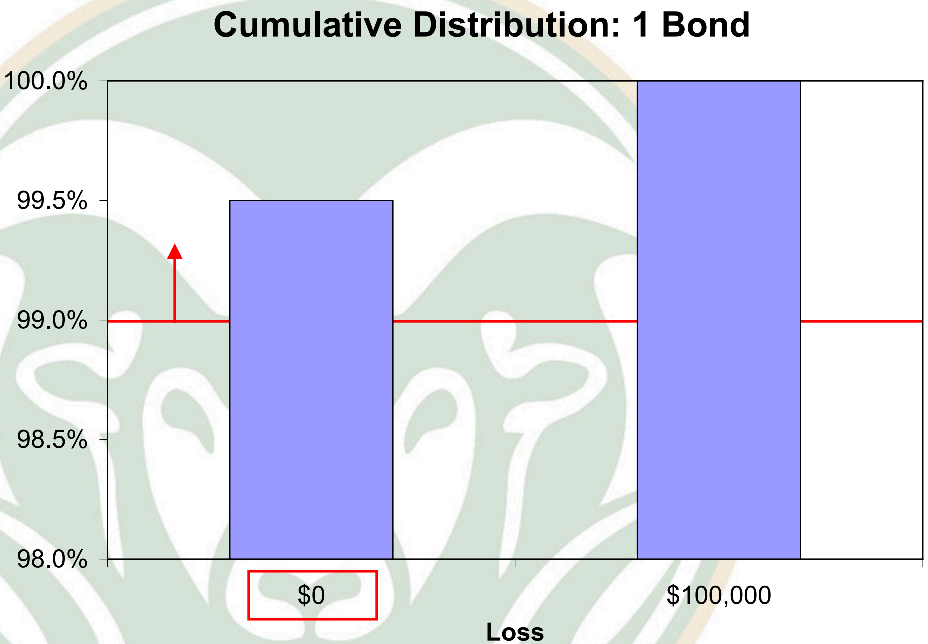
Why VAR is not Necessarily Subadditive

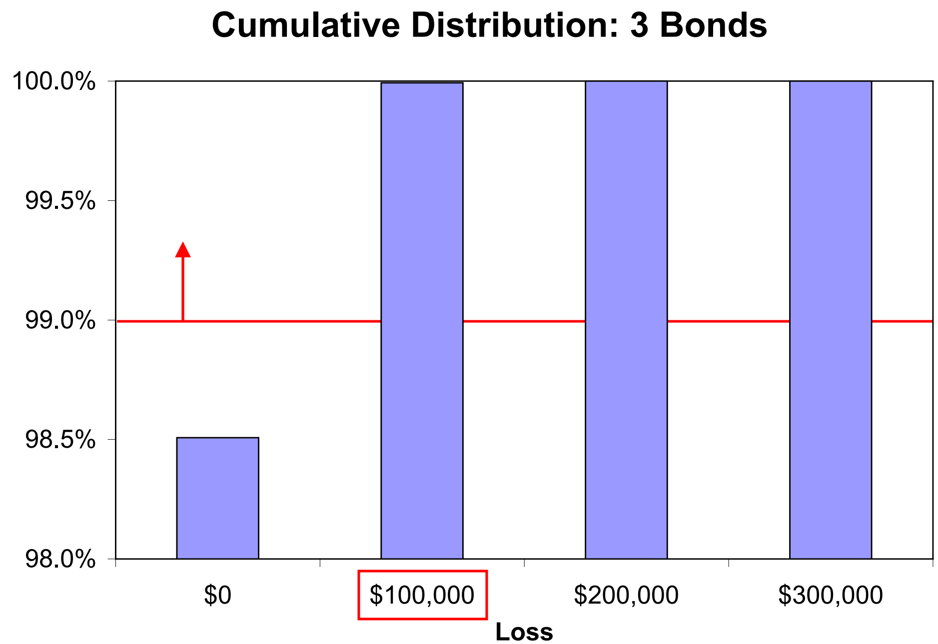
- Consider an investment in a corporate bond with face value of \$100,000 and default probability of 0.5%; the portfolio has 3 such bonds, with independent defaults
- For each bond, returns are -\$100,000 with probability of 0.5% and \$0 with prob of 99.5%
- Joint loss distribution is:

State	Probability	Payoff
No default	$0.995^3 = 0.985075$	\$0
1 default	$3*0.005*0.995^2 = 0.014850$	-\$100,000
2 defaults	$3*0.005^2 *0.995 = 0.000075$	-\$200,000
3 defaults	$0.005^3 = 0.0000001$	-\$300,000

Computation of VAR

- Lowest loss (as positive value) such that the probability of losing more is at least 99%
- VAR for 1 bond is $0
- VAR for 3-bond portfolio is $100,000





Non-Subadditive VAR

- Adding up the 3 VARs gives \$0
- Portfolio VAR=\$100,000
- Thus $\rho(\Sigma W) > \Sigma\ \rho(W)$: VAR is not subadditive
- This may be an issue for concentrated portfolios, or at the level of an option trader
- This is less of an issue, however, for large portfolios
  - most empirical work shows little difference in classifications based on VAR or ETL
  - no bank reports ETL

Problem of VAR

- Does not provide information of the actual values which might be expected in the extremes, only the value associated with a given percentile
  - A threshold value of loss yet not a expected value of loss
  - Focuses on the “good states” (the 99 days) rather than the “bad scenarios” (that 1 day)
- Moral hazard
  - Traders/managers “game” the performance target as extreme tail losses do not affect VaR

More on VaR Measure of Risk

- Why still use VaR?
  - Coherent for elliptical distributions
  - Central limit theorem for large portfolios
- More reasons for the popularity
  - A "common" measure across positions and risk factors
  - "Aggregate" and "holistic": taking account of different risk factors
  - "Probabilistic": as opposed a fixed number
  - A good "unit of measure"
- Other risk measures?

Quantile-Based Risk Measures

- What is Quantile-based risk measure (QBRM)?
- Why QBRM?
  - Try to maintain the strengths of VaR
    - Based on the tail of the distribution
    - Probabilistic, universal measure
  - But overcome some major problems
    - Coherent
    - Gives information on the tail events
    - Other considerations

Expected shortfall/tail loss (Conditional VaR)

- Take a summary measure of the tail area – average of the worst 1- α losses
- Discrete case: $ES_\alpha = \frac{1}{1-\alpha}\sum_{p=\alpha}^{1}(pth\ worst\ outcomes)\times(respective\ probability)$
- Continuous case: $ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 F^{-1}(p)dp$
- "Equivalently": $E[X \mid X > q_\alpha(X)]$
- Other names: expected tail loss, conditional VaR, etc.

Expected shortfall (Conditional VaR)

- Coherence of ES
  - Consider the discrete case
    - $ES_\alpha(X) + ES_\alpha(Y)$ = Mean of $N\alpha$ worst cases of X +Mean of $N\alpha$ worst cases of Y $\geq$ Mean of $N\alpha$ worst cases of (X+Y) = $ES_\alpha(X+Y)$
  - For the continuous case, take to the limit as $N \rightarrow \infty$

- Coherent risk measures as a result of scenario analyses
- Any shortcomings of ES?

Expected shortfall (Conditional VaR)

- Expected loss conditional on going out in the left tail
- This is also called "Conditional VaR"

- Advantages
  - Better information on possible tail losses
  - Some better properties (sub-additive)
- Disadvantages
  - Sensitive to outliers
  - Difficult to estimate (for high confidence numbers)
  - More difficult to explain

Example : Calculation of 1-day, 99% VaR for a Portfolio on Sept 25, 2008

- Equal Weight Model

Simulation Approach

	DJIA	FTSE 100	CAC 40	nikkei 225
DJIA	1			
FTSE 100	0.489105943	1		
CAC 40	0.495709627	0.918108	1	
nikkei 225	-0.061899208	0.200942	0.210951	1

	DJIA	FTSE 100	CAC40	Nikkei 225
Return	0	0	0	0
Gross Return	1	1	1	1

Portfolio Loss	0
Forecast Name	Portfolio
Standard Deviation	94.22
Variance	8,878.11
1%	-211.44
One-Day 99% VaR	211.44
Conditional Shortfall	0
Conditional VAR	0
Forecast Name	Conditional VAR
Mean	-250.72
One-Day 99% CVaR	250.72

	DJI	FT	C	Ni
DJIA (Equal	1.000			
FTSE 100 (E	0.489	1.000		
CAC40 (Equ	0.496	0.918	1.000	
Nikkei 225	-0.062	0.201	0.211	1.000

Example : Calculation of 1-day, 99% VaR for a Portfolio on Sept 25, 2008

- EWMA

Simulation Approach

	DJIA	*FTSE 100*	*CAC 40*	*nikkei 225*
DJIA	1			
FTSE 100	0.611	1		
CAC 40	0.629	0.971	1	
nikkei 225	-0.113	0.409	0.342	1

	DJIA	FTSE 100	CAC40	Nikkei 225
Return	0	0	0	0
Gross Return	1	1	1	1

Portfolio Loss	0

Forecast Name	Portfolio Loss
Standard Deviation	204.50
Variance	41,819.90
1%	-477.28

One-Day 99% VaR	477.28
Conditional Shortfall	0
Conditional VAR	0

Forecast Name	Conditional VAR
Mean	-566.81

One-Day 99% CVaR	566.81

	Ret	FT:	C	Ni
Return (EWMA)	1.000			
FTSE 100 (EWMA)	0.611	1.000		
CAC40 (EWMA)	0.629	0.971	1.000	
Nikkei 225 (EWMA)	-0.113	0.409	0.342	1.000

VaR vs. Conditional VaR

- VaR is the loss level that will not be exceeded with a specified probability
- Expected Shortfall (or C-VaR) is the expected loss given that the loss is greater than the VaR level
- Although expected shortfall is theoretically more appealing, it is VaR that is used by regulators in setting bank capital requirements